Ph.D. QUALIFYING EXAMINATION
DEPARTMENT OF PHYSICS AND ASTRONOMY
WAYNE STATE UNIVERSITY

PART II

MONDAY, May 5, 2014
9:00 AM — 1:00 PM

ROOM 245 PHYSICS RESEARCH BUILDING

INSTRUCTIONS: This examination consists of six problems, each worth 10 points. Use a separate booklet for each problem. Write the following information on the front cover of each booklet:

1. Your special ID number that you received from Delores Cowen;
2. The problem number and the title of the exam (*i.e.* Problem 1, Part II).

Please make sure your answers are dark and legible.

Do NOT write your name on the cover or anywhere else in the booklet!

1. (**10 points**) A very tall cylinder with a cross-section of 1 m^2 is set vertically from the ground. The air temperature inside the tube is uniformly 300 K. The air particle density at the ground level is $n_0 = 2.69 \times 10^{25}$ m^{-3}, and the molar mass of air is $M = 29$ g/mol. Assume ideal gas law for the air.

   (a) Calculate the air pressure as a function of height Z. (7 pts)

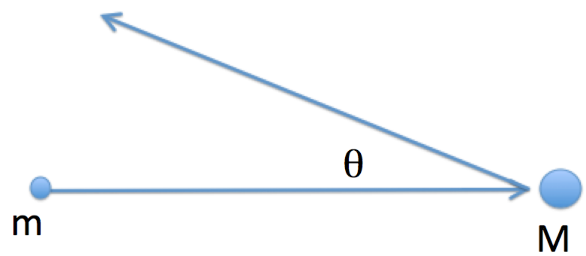
   (b) Find the total number of air particles inside the tube. (3 pts)

2. (**10 points**) In Rutherford back-scattering, atoms are identified by shooting an α particle of mass m and initial kinetic energy E onto a rest target of unknown mass M. The particle is then detected at an angle θ from the back-scattering direction, with measured energy E_1.

   (a) Write down the energy and momentum conservation equations. (2 pts)

   (b) Solve for M as a function of the given quantities. (5 pts)

   (c) If $m = 4$ atomic units (a.u.), $E = 2$ MeV, $E_1 = 0.71$ MeV, and $\theta = 10\ mrad$, find which atom is being bombarded. (3 pts)



3. (**10 points**) Positronium is a bound state of an electron and a positron. Both the energy levels and wave functions of positronium are similar to those of hydrogen.

(a) What is the normalized wave function of the $1s$ ground state for positronium (without considering spin)? Use the result of $1s$ ground state for hydrogen: $\psi_{100} = \exp(-r/a_0)/\sqrt{\pi a_0^3}$, where the Bohr radius $a_0 = 4\pi\epsilon_0\hbar^2/(m_e e^2)$ with m_e the electron mass. (2 pts)

(Hint: use the reduced mass.)

(b) In positronium the spin operators of electron and positron are given by $\hat{\mathbf{S}}_e$ and $\hat{\mathbf{S}}_p$. What are the allowed values of quantum numbers for the total spin S and the corresponding z-component S_z? (2 pts)

(c) The $1s$ state of positronium is subjected to a hyperfine interaction

$$\hat{H}' = -\beta \hat{\boldsymbol{\mu}}_e \cdot \hat{\boldsymbol{\mu}}_p \delta(\mathbf{r}),$$

where $\hat{\boldsymbol{\mu}}_e = -\frac{e}{m_e c}\hat{\mathbf{S}}_e$ and $\hat{\boldsymbol{\mu}}_p = \frac{e}{m_e c}\hat{\mathbf{S}}_p$ are the magnetic moments and β is a constant. Use first-order perturbation theory to calculate the change of system energy. You need to consider each spin state obtained in (b). (6 pts)

4. (**10 points**) A two-level system of $N = n_1 + n_2$ particles is distributed among two eigenstates 1 and 2 with eigenenergies E_1 and E_2 respectively. The system is in contact with a heat reservoir at temperature T. If a single quantum emission into the reservoir occurs, population changes $n_2 \to n_2 - 1$ and $n_1 \to n_1 + 1$ take place in the system.

(a) Give Boltzmann's statistical definition of entropy and present its physical meaning briefly but clearly. (1 pt)

(b) For $n_1 \gg 1$ and $n_2 \gg 1$ and using the result of (a), obtain the expression for the entropy change of the two-level system in terms of n_1 and n_2. (3 pts)

(c) Find the entropy change of the reservoir. (3 pts)

(d) From results of (a) and (b), derive the Boltzmann relation for the ratio n_2/n_1. (3 pts)

5. (**10 points**) A particle of mass m moves on a paraboloid $z = \alpha(x^2 + y^2)$, subjected to the force of gravity mg directed towards the negative z direction.

   (a) Derive the particle Lagrangian. (5 pts)

   (b) Is angular momentum conserved in this system? Discuss. (2 pts)

   (c) Find the condition for which the particle follows a circular motion. (3 pts)

6. (**10 points**) Consider a sphere with radius R made of a linear homogeneous dielectric material with permittivity ϵ, concentrically surrounding a solid conducting sphere of radius a (see the figure below). Charges are distributed (glued) on the outer dielectric surface (i.e., the spherical surface of radius R) with density $\sigma = \sigma_0 + \sigma_1 \cos\theta$, where θ is the polar angle.

(a) Find the electric field E and potential V of this configuration at a distance $r \gg R$. Use simple physical arguments and approximations. Don't solve Laplace equation for this part. (3 pts)

(b) Set up Laplace equation for the potential in spherical coordinates and specialize to the case of azimuthal-angle ϕ independent solutions. Introduce separation of variables and write down the most general form of the solution which has no ϕ dependence. Write down all the boundary conditions on the potential and the electric field that are needed to find the potential everywhere in this problem. Here you may express the induced charge density on the conductor as $\sigma_c(\theta)$. (3 pts)

(c) Use the results of (b) to find the potential V everywhere for the given configuration. (4 pts)