Ph.D. QUALIFYING EXAMINATION
DEPARTMENT OF PHYSICS AND ASTRONOMY
WAYNE STATE UNIVERSITY

PART I

FRIDAY, May 2, 2014
9:00 AM — 1:00 PM

ROOM 245 PHYSICS RESEARCH BUILDING

INSTRUCTIONS: This examination consists of six problems, each worth 10 points. Use a separate booklet for each problem. Write the following information on the front cover of each booklet:

1. Your special ID number that you received from Delores Cowen;
2. The problem number and the title of the exam (*i.e.* Problem 1, Part I).

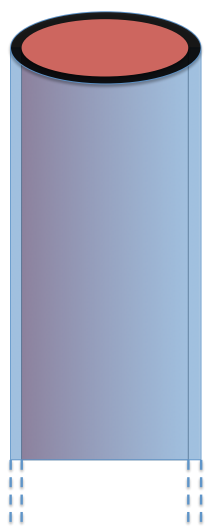
Please make sure your answers are dark and legible.

Do NOT write your name on the cover or anywhere else in the booklet!

1. (**10 points**) Two very long cylinders with radii R and $R + \delta R$ ($\delta R \ll R$) are placed concentrically (one inside the other) and stood upright as shown in the figure below. Both cylinders are made of insulating material with permittivity ϵ and permeability of vacuum. The inner cylinder is solid while the outer one is a shell. Positively charged particles are then filled and moved inside the narrow cylindrical shell along the edge in a cyclonic velocity pattern $\vec{v} = v_z\hat{k} + v_\phi\hat{\phi}$. These particles are moved by non-electromagnetic forces and have a volume charge density ρ. The gap between the cylinders is narrow enough that one can approximate this shell as an effective charged surface.

   (a) Calculate the effective surface charge density and current density. Ignore any Lorentz force or time dependent effects. (4 pts)

   (b) Calculate the E and B field everywhere for this configuration. Ignore any Lorentz force or time dependent effects. (6 pts)



2. (**10 points**) Two particles of masses m_1 and m_2 are initially at rest and separated by a distance a. They fall towards each other due to a force $-k/r^2$. Use the reduced mass to solve the followings:

   (a) Calculate the energy of the system. (2 pts)

   (b) From the system energy, derive an equation for the time t_0 that it takes for the two particles to fall onto each other. (5 pts)

   (c) Solve the equation to get t_0, and compare it to the orbital period t_1 if the two particles were orbiting each other in a circular orbit. (3 pts)

   (Hint: You might need to use the change of variable $u^2 = 1 - r/a$ to perform the integration.)

3. (**10 points**) Consider a particle of mass m in a one-dimensional potential

$$V(x) = \begin{cases} 0, & 0 \leq x \leq a, \\ \infty, & x < 0 \text{ or } x > a. \end{cases}$$

At $t = 0$ the particle's initial wave function is given by

$$\Psi(x, t = 0) = \sqrt{\frac{32}{5a}} \sin(2\pi x/a) \cos^2(\pi x/a).$$

(a) What is the expectation value of the particle energy at $t = 0$? (4 pts)

(b) What is the wave function at a later time $t = T$? What is the expectation value of the particle energy at this time? (3 pts)

(c) What is the probability that the particle is found in the left half of the potential well (i.e., $0 \leq x \leq a/2$) at $t = T$? Show your calculation steps. (3 pts)

4. (**10 points**) A cylindrical rod of length l insulated on the lateral surface is initially in contact at one end with a reservoir at temperature $T_H = 400$ K and the other end with another reservoir at a lower temperature $T_C = 300$ K. The temperature within the rod initially varies linearly with position x according to $T_i(x) = T_H - (\frac{T_H - T_C}{l})x$. The rod is then isolated from the two reservoirs simultaneously and eventually it comes to a final equilibrium at a temperature T_f. The mass and the specific heat of the rod are $m = 1$ kg and $c_p = 385$ J/(kg·K) respectively.

   (a) Considering the rod thermally isolated after being disconnected from the reservoirs, express T_f in terms of T_H and T_C. (1 pt)

   (b) For a segment dx located at x, evaluate the entropy change dS from the moment of disconnecting the reservoirs to the moment of reaching T_f. (3 pts)

   (c) Evaluate the entropy change for the whole rod from the moment of disconnecting the reservoirs to the moment of reaching T_f. (3 pts)

   (d) If only the cold reservoir is removed instead and an equilibrium is reached, evaluate the entropy change in the rod. (3 pts)

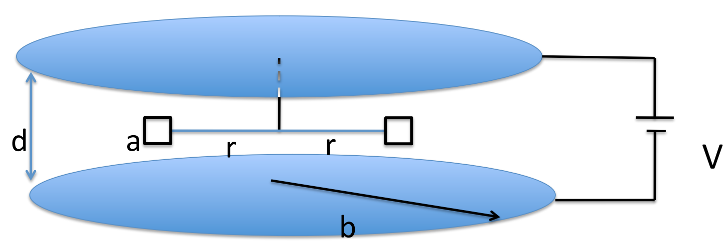
5. (**10 points**) Consider a circular-shaped, parallel plate capacitor which is being slowly charged (see the figure below). Two conducting square-shaped wire loops (with side a and a small resistance Ω_2 for each side) are attached to an insulating rod of length $2r$. The rod is hung at the middle from the center of the top capacitor plate by an insulating thread. The entire hanging assembly is free to move and rotate. The rod can expand or contract lengthwise with an elastic restoring force $F = -k\Delta r$. Assume the maximum voltage of the battery to be V.

(a) Assuming that the hanging bracket assembly within the capacitor has a minimal effect on the overall charging of the capacitor, calculate the potential drop across the capacitor plates as a function of time t, maximum voltage V, the resistance in the outer circuit R and the capacitance C. (2 pts)

(b) Use this time-dependent potential to find the time-dependent electric field E and the time-dependent magnetic field B in the capacitor (again ignoring the effect of the hanging assembly within). (3 pts)

(c) From the expression of the space-time dependence of the B field, calculate the current in the square loops and hence the net Lorentz force on the squares when they are at a distance r from the center. Ignore any transient currents from the E field. (3 pts)

(d) Quantitatively describe what happens to the hanging bracket assembly as the capacitor begins to charge up. (2 pts)



6. (**10 points**)

(a) Prove the three-dimensional virial theorem $2\langle K\rangle = \langle \mathbf{r}\cdot\boldsymbol{\nabla} V\rangle$ for any stationary state of a nonrelativistic particle, where K is the kinetic energy and V is the potential energy. (6 pts)

(Hint: use the relation $d\langle Q\rangle/dt = (i/\hbar)\langle[\hat{H},\hat{Q}]\rangle + \langle \partial\hat{Q}/\partial t\rangle$ and set $Q = \mathbf{r}\cdot\mathbf{p}$.)

(b) Based on the result of (a), show that for a nonrelativistic particle of mass m in a logarithmic central potential $V(r) = \alpha\ln(r/r_0)$, where α and r_0 are constants and $r = |\mathbf{r}|$, all eigenstates have the same mean-squared velocity $\langle v^2\rangle$. Find this mean-squared velocity. (4 pts)