Ph.D. QUALIFYING EXAMINATION
DEPARTMENT OF PHYSICS AND ASTRONOMY
WAYNE STATE UNIVERSITY

PART I

MONDAY, MAY 9, 2011
9:00 AM — 1:00 PM

ROOM 245 PHYSICS RESEARCH BUILDING

INSTRUCTIONS: This examination consists of six problems each worth 10 points. Use a separate booklet for each problem. Write the following information on the front cover of each booklet:

1. your special ID number that you received from Delores Cowen,
2. the problem number and the title of the exam (*i.e.* Problem 1, Part I).

Please make sure your answers are dark and legible.

Do NOT write your name on the cover or anywhere else in the booklet!

1. **(10 points):** General Questions

(a) (2 points) Consider an object with mass m and volume V submerged in a fluid of density ρ. The buoyant force is the interaction force between the object and the fluid the object is submerged in.

i. What is the buoyant force (magnitude and direction) when the combined system of object and fluid is at rest on the surface of the Earth?

ii. What is the buoyant force when the combined system of object and fluid is at rest in a space craft floating freely in space far from any other gravitating object.

Quantum phenomena are often negligible in the macroscopic world. Show that this is true in the following two examples.

(b) (4 points) Estimate the amplitude of the zero-point oscillation for a pendulum of length $\ell = 1\,\mathrm{m}$ and mass $m = 1\,\mathrm{kg}$.

(c) (4 points) Estimate the tunneling probability for a ball of mass $m = 1\,\mathrm{kg}$ colliding at a speed $v = 1\,\mathrm{m/s}$ with a rigid obstacle of height $H = 1\,\mathrm{m}$ and width $W = 1\,\mathrm{cm}$.

2. **(10 points):** A suitcase of mass m is placed (with zero speed) on a level, straight conveyer belt that moves with constant speed v. The suitcase slides on the belt before it eventually moves together with the belt at speed v. The kinetic coefficient of friction between the belt and the suitcase is μ_k.

   (a) (3 points) For what time interval does the suitcase slide on the belt?

   (b) (3 points) Calculate the total work done by the frictional force.

   (c) (4 points) Calculate the total work done by the motor driving the belt.

3. **(10 points)**: A very long solenoid of n turns per unit length carries a current which increases uniformly with time, $I(t) = Kt$. Here K is a positive constant.

   (a) (2 points) Calculate the magnetic field inside the solenoid at time t (neglect retardation).

   (b) (2 points) The time varying magnetic field produces an electric field. Working in cylindrical coordinates (r, θ, z), examine the symmetries in the problem to determine which coordinates the electric field can depend on.

   (c) (2 points) Calculate the electric field inside the solenoid.

   (d) (4 points) Consider a cylinder of length l and radius equal to that of the solenoid and coaxial with the solenoid. Find the rate at which energy flows into the volume enclosed by this cylinder using the Poynting vector and show that it is equal to $\frac{d}{dt}\left(\frac{1}{2}lLI^2\right)$, where L is the self-inductance per unit length of the solenoid.

4. **(10 points):** Einstein's model for lattice vibrations

(a) (3 points) Consider a simple harmonic oscillator of mass m coupled to a spring with spring constant k. Apply quantum statistics and derive an expression for the energy $U(m, k, T)$ of the simple harmonic oscillator at temperature T.

(b) (5 points) Einstein treated the atomic oscillations in a solid using a model of a set of N uncoupled simple harmonic oscillators , each with mass m and spring constant k. Calculate the heat capacity for the system of N uncoupled, simple harmonic oscillators.

(c) (2 points) Find the temperature dependence of the heat capacity in the high and low temperature limits. Comment on your results.

5. **(10 points)**: A particle of mass m can move only in one dimension. It is originally connected to two identical springs, one on each side. Each spring has a spring constant k and relaxed length a. The springs are fixed to the points $x = +a$ and $-a$, respectively, so that at $x = 0$ the particle experiences no force classically. Treat the particle quantum mechanically.

   (a) (2 points) What are the energy eigenvalues of the particle? What is the ground state energy of the particle?

   (b) (2 points) What is the ground state wave function of the particle?

   (c) (6 points) One of the springs is suddenly disconnected from the particle while it is in the ground state. What is the probability of the particle remaining in the (new) ground state?

the ground state wave function for a simple harmonic oscillator of mass m and spring constant k is

$$\psi_0(x) = \frac{\sqrt{\alpha}}{\pi^{1/4}} e^{-\alpha^2 x^2/2}$$

where

$$\alpha = \left(\frac{mk}{\hbar^2}\right)^{1/4}.$$

6. **(10 points)**: A small block of mass m_b is released from rest and slides down a wedge of mass m_w ($> m_b$) and angle of inclination θ. The wedge rests on a flat, horizontal table. All surfaces are smooth and without friction.

   (a) (5 points) Calculate the acceleration of the block with respect to the wedge.

   (b) (5 points) Calculate the acceleration of the wedge.

Ph.D. QUALIFYING EXAMINATION
DEPARTMENT OF PHYSICS AND ASTRONOMY
WAYNE STATE UNIVERSITY

PART II

WEDNESDAY, MAY 11, 2011
9:00 AM — 1:00 PM

ROOM 245 PHYSICS RESEARCH BUILDING

INSTRUCTIONS: This examination consists of six problems each worth 10 points. Use a separate booklet for each problem. Write the following information on the front cover of each booklet:

1. your special ID number that you received from Delores Cowen,
2. the problem number and the title of the exam (*i.e.* Problem 1, Part II).

Please make sure your answers are dark and legible.

Do NOT write your name on the cover or anywhere else in the booklet!

1. **(10 points)**: Consider a classical particle of mass m moving in two dimensions in a potential

$$V(x,y) = -\frac{1}{2}kx^2 + \frac{1}{2}\lambda_0 x^2 y^2 + \frac{1}{4}\lambda_1 x^4, \qquad k, \lambda_0, \lambda_1 > 0 \ .$$

   (a) (3 points) At what point(s) (x_0, y_0) is the particle in stable equilibrium?

   (b) (7 points) Construct the Langrangian. Calculate the normal mode frequencies for small oscillations about the position(s) of stable equilibrium.

2. **(10 points):** An atomic clock is carried on a satellite in a circular orbit, 300 km above Earth's equator.

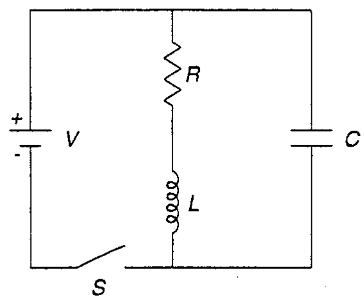
   (a) (4 points) Determine the satellite's speed and period of orbit (as seen from the Earth).

   (b) (2 points) Consider effects due to special relativity when answering the following. According to an observer on Earth, does the clock on the satellite run faster or slower than an identical clock at rest on the Earth's surface.

   (c) (4 points) What is the time difference accumulated each time the satellite completes a rotation? Ignore the effect of Earth's rotation.

   Treat the Earth as a sphere of radius 6370 km and mass 5.98×10^{24} kg.

3. **(10 points):** Consider the circuit drawn below. The switch S is closed for a long time to allow the circuit to reach a steady state before it is opened at time $t = 0$.



(a) (5 points) Calculate the current through the inductor as a function of time ($t > 0$) when $R^2C/4L > 1$.

(b) (5 points) Calculate the current through the inductor as a function of time ($t > 0$) when $R^2C/4L = 1$.

4. **(10 points):** In a certain representation, the physical observables M_1 and M_2 are represented by the matrices:

$$M_1 = \frac{m_1}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \quad \text{and} \quad M_2 = m_2 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

respectively.

(a) (6 points) When the state of the system is

$$|\psi\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

the physical observable M_1 is measured. What are the possible outcomes of the measurement? Calculate the probability of obtaining each of the possible outcomes.

(b) (4 points) After the measurement of M_1 is done and the result of *zero* is obtained, the observable M_2 is measured. What are the possible outcomes of the measurement and the corresponding probability for each of the possible outcomes?

5. **(10 points):** A particle is placed in a three dimensional infinite square well potential field $V(x, y, z)$ where

$$V(x,y,z) = \begin{cases} 0 & \text{for } -\frac{a}{2} < \{x, y, z\} < \frac{a}{2} \text{ and} \\ \infty & \text{otherwise.} \end{cases}$$

(a) (2 points) How many ground states does the particle have? Give the eigenenergies and the normalized eigenfunctions for each ground state.

(b) (3 points) How many first excited states does the particle have? Give the eigenenergies and the normalized eigenfunctions for each first excited state.

(c) (5 points) Now a small perturbation $H' = \alpha xyz^2$ is applied, where α is a real constant. Calculate, to first order in α, the modification to the energy of the particle for each of the states in parts (a) and (b).

6. **(10 points):** To find the ratio C_P/C_V for an ideal gas, one can use the following method.

A certain amount of gas with initial temperature T_0, pressure P_0, and volume V_0 is heated by an electric current flowing through a resistor for a time interval t. The experiment is done twice: first at constant volume V_0 with the pressure change from P_0 to P_f recorded, and second at constant pressure P_0 with the volume change from V_0 to V_f recorded. The time interval t is the same in both experiments.

Express the ratio C_P/C_V in terms of the measured values of the initial and final pressures and volumes.