Ph.D. QUALIFYING EXAMINATION
DEPARTMENT OF PHYSICS AND ASTRONOMY
WAYNE STATE UNIVERSITY

PART I

Monday, May 7, 2007
9:00 A.M. — 1:00 P.M.

ROOM 245 PHYSICS BUILDING

INSTRUCTIONS: This paper contains six problems. The first four problems are worth 10 points per problem and the last two are worth 15 points. You are to solve all six using a separate booklet for each problem. On the front cover of each booklet, you must write the following information.

1) Your special ID number that you obtained from Delores Cowen
2) The problem number and the title of the exam (i.e. problem #1, part #1)
3) Please press hard and make your answers legible.

Please do NOT write your name on the cover or anywhere else in the booklet!

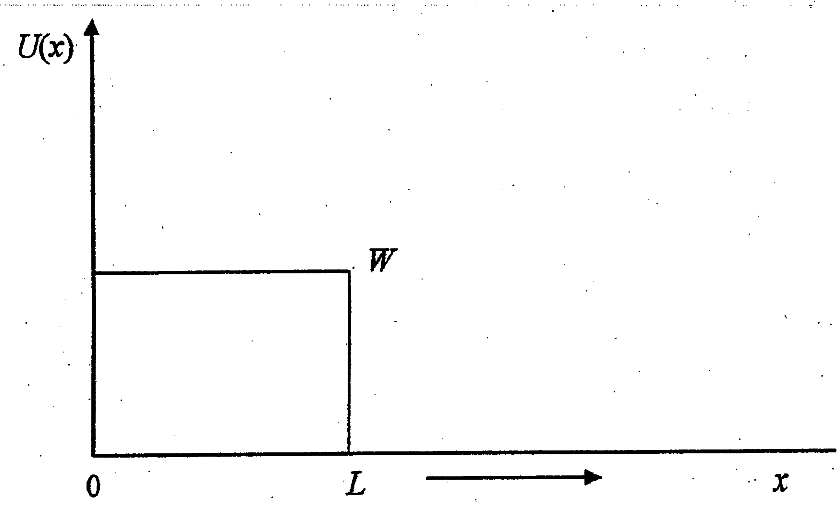
1. (10 pts). A car is stuck in the mud. In his efforts to move the car, the driver splashes mud from the rim of a tire of radius R spinning at a speed v. Neglecting the resistance of the air, find the maximum height mud can rise *above ground.*

2. (10 pts). Consider a particle of mass m in one dimension (call it the x direction). The potential it experiences (see the figure) is infinity for negative x and zero for $x > L$. There is a constant potential $W > 0$, running from $x = 0$ to $x = L$. W is equal to five times the kinetic energy, E, the particle would have if it were detected in the positive region of x moving toward the barrier: $W = 5E$. In the region $x > L$, where the potential is zero, the particle has a speed $v = h/2mL << c$ [the speed of light]. Consider the solution of the appropriate quantum mechanical time-independent wave equation with this energy E.

a) (4 pts). Show the form of the wavefunction, corresponding to this solution, in every region of x. Use appropriate boundary conditions to determine the wavefunction in terms of m and L, up to an overall multiplicative constant.

b) (4 pts). From your result in (a), calculate the ratio of the probability that a particle in that state could be found moving away from the barrier in the region $x > L$ to the probability of finding it moving towards the barrier from that region, i.e. the reflection coefficient (call it R).

c) (2 pts). Give a physical argument that automatically determines the reflection coefficient you calculated in (b).



3. (10 pts). A classical model of electrical conductivity in metals assumes a uniform distribution of free electrons. Assume that there are N such free electrons in a volume V. The motion of a single electron is then described by Newton's 2nd law

$$m\frac{d\vec{v}}{dt} = e\vec{E} - \beta\vec{v}$$

where $\vec{E}$ is the electric field and β is a damping (friction) coefficient in the material.

(a) (2 pts). Starting from Newton's second Law, derive the differential equation for the current density vector in the material.

(b) (3 pts). Find the current density in the case of constant current flow and thus find the DC conductivity (σ) of the material defined by $\vec{j} = \sigma\vec{E}$.

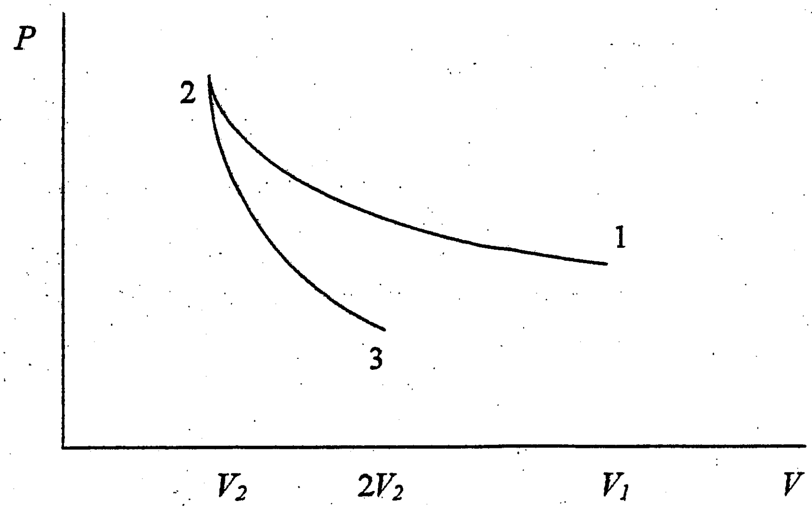
(c) (5 pts). Now suppose an oscillating electric field of amplitude $\vec{E}_0$ and frequency ω is applied, of the form $\vec{E} = \vec{E}_0 e^{-i\omega t}$. Find the steady state oscillating current and thus the conductivity of the material in this case and derive the frequency dependence of the phase shift between the current density and the applied electric field.

4. (10 pts). An ideal gas of total N molecules is compressed at constant temperature T_1 from volume V_1 to volume V_2.

a) (3pts). Find the work done on the gas. Is the heat absorbed or rejected by the gas in this process?

b) (5pts). The gas then expands adiabatically to volume $V_3 = 2V_2$. Using the fact that the entropy in this process is conserved, derive the expression for the final temperature of the gas T_f.

c) (2pts). What is T_f for air (considered to be an ideal gas) initially at room temperature $T_1 = 300K$?

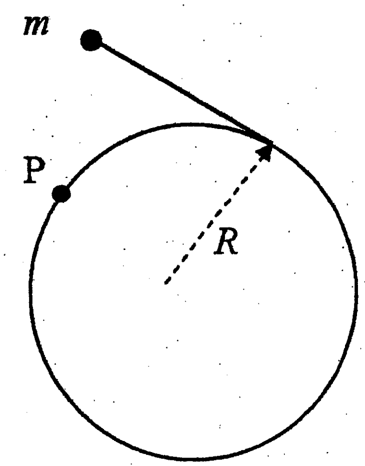


5. (15 pts). A point mass, m, moving on a horizontal frictionless table, is attached with a weightless cord to a fixed cylinder of radius R. Initially the cord is wound around the cylinder in such a way that the mass is touching the cylinder at point P. An impulsive force given to the mass directed radially, giving the mass an initial velocity v_0, so that the cord starts to unwind (see the figure).

a) (3 pts). Is the angular momentum conserved? Why? Is the energy conserved? Why?

b) (8 pts). Write down the equation of motion in terms of suitable generalized coordinate(s) and solve this equation.

c) (4 pts). Find the angular momentum of the mass about the cylinder axis.



6. (15 pts.). The x-y plane defines the interface between two infinite volumes of uniform dielectric. The dielectric constant in the region $z > 0$ is $\varepsilon = \varepsilon_1$ and is $\varepsilon = \varepsilon_2$ in the region $z < 0$. A single point charge q is embedded at the point $x = 0$, $y = 0$, $z = d$.

(a) (6 pts). From Maxwell's equations as applied to these dielectric media, derive the boundary conditions satisfied by the transverse component of the electric field, $\bar{E} \cdot \hat{t}$ and the normal component of the electric displacement, $\bar{D} \cdot \hat{n}$ at $z = 0$ interface.

(b) (9 pts). Determine the electric potentials in cylindrical (or other appropriate coordinates) $\phi_1(\rho, z)$ for $z > 0$ and $\phi_2(\rho, z)$ for $z < 0$.

Hint: For part b) you may find it useful to apply the method of image charges and the boundary conditions found in part a).

Ph.D. QUALIFYING EXAMINATION
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WAYNE STATE UNIVERSITY

PART II

Wednesday, May 9, 2007
9:00 A.M. — 1:00 P.M.

ROOM 245 PHYSICS BUILDING

INSTRUCTIONS: This paper contains six problems. The first four problems are worth 10 points per problem and the last two are worth 15 points. You are to solve all six using a separate booklet for each problem. On the front cover of each booklet, you must write the following information.

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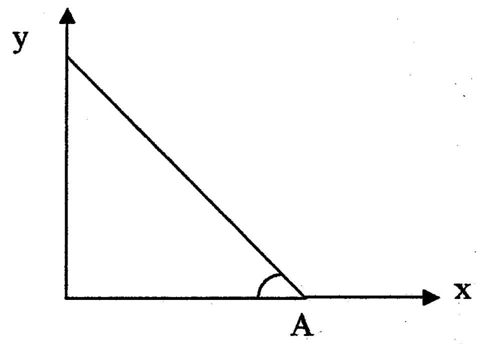
1. (10 pts). Two spaceships, whose coordinate systems are aligned with one another, are passing each other in a region very far from any other objects. Spaceship 1 ($S^{(1)}$) sees spaceship 2 ($S^{(2)}$) passing it with speed 0.6c along the positive x-axis. They each set their stopwatches to zero as their origins pass. $S^{(2)}$ has a triangle painted on its hull that is measured by its crew and found to be the isosceles right triangle oriented as shown below:



The angle at A is measured by the crew to be 45°, as is true in any isosceles triangle.

(a) (2 pts). What angle will the crew of $S^{(1)}$ measure at vertex A?

Suddenly a light flashes for an instant. A crew member on $S^{(1)}$ determines that the location of the flash was: $x^{(1)} = 300$ km, $y^{(1)} = 400$ km, and $z^{(1)} = 500$ km; and the time of the occurrence was $t^{(1)} = 5/3$ ms.

(b) (2 pts). Find the location and time that an observer in $S^{(2)}$ measures for the same flash.

(c) (2 pts). Calculate the invariant distance in 4d space-time (interval) between the location of the flash and the origins' crossing.

(d) (2 pts). Where is $S^{(2)}$, as measured by $S^{(1)}$, at the time of the flash?

(e) (2 pts). Where is $S^{(1)}$, as measured by $S^{(2)}$, at the time of the flash?

2. (10 pts). Consider N distinguishable spins at a temperature T. The spins are in a chain where each spin can only interact with the nearest neighbor with the interaction energy $-\varepsilon$ when the two spins are parallel to each other or $+\varepsilon$ when the spins are anti-parallel (the spins have the values ± 1).

a) (4 pts). What are the possible total energy states for this system for $N = 3$? What are the multiplicities of each state?

b) (6 pts). The partition function for N spins for this system is $Z = (e^{-\varepsilon/\tau} + e^{\varepsilon/\tau})^{N-1}$, where $\tau = K_B T$. Using this function, calculate the average energy U of the system and plot it as a function of ε/τ. What are the values of U at $T \to 0$ and at $T \to \infty$?

3. (10 pts). An infinitely long solenoid of radius R is tightly wound with wire at N turns per unit length. The solenoid is aligned along the z-axis and the current flows in the direction of the azimuthal unit vector $\hat{\phi}$. Assume that the current I in the winding wire increases with time at a constant rate $I(t) = I_0 + \alpha t$ where $dI/dt = \alpha > 0$.

(a) (6 pts). Find the magnitude and direction of the electric and magnetic fields everywhere in space.

(b) (4 pts). Find the magnitude and direction of the Poynting vector at time t everywhere in space.

4. (10 pts). For a normalizable solution of the time-dependent Schrödinger equation with the potential $V(r)$ and $\langle \vec{r} \rangle$ the expectation value of a position operator

(a) (5 pts). Calculate $\frac{d}{dt}\langle \vec{r} \rangle$ and express your result in terms of the $\vec{p}$ operator.

(b) (5 pts). Calculate $\frac{d^2}{dt^2}\langle \vec{r} \rangle$, and show how it is related to Newton's laws.

5. (15pts). Consider two spin-½ free particles, which are not identical. Particle 1 has spin up in the z-direction, and particle 2 has spin down in the z-direction. For your information, the Pauli matrices are:

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

(a) (3 pts). Verify that there is a non-zero probability of measuring the total spin of this system to be $S_{TOTAL} = 1$. Find that probability. For this part, you may use $\uparrow$ for particle 1 , $\downarrow$ for particle 2. With this notation, you can represent the spin state as $\uparrow\downarrow$, where the first entry refers to particle 1 and the second entry to particle 2.

(b) (3 pts). For a single spin-½ particle write matrices S_x, S_y, and S_z, in a basis where the column vector $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ represents the state of a particle with spin up in the z-direction, and the column vector $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ the state of a particle with spin down in the z-direction.

(c) (3 pts). Find a column vector that represents a particle with spin up in the x-direction, and a column vector that represents spin down in the x-direction.

(d) (3 pts). Suppose that particle 1 (which has spin up in the z-direction) is sent thorough a device that measures its spin in the x-direction. From your answers in (c), find the probability that it will be measured to have spin down in the x-direction.

(e) (3 pts). Suppose particle 2 (which has spin down in the z-direction) is a positron. It is sent into a region where there is a uniform unchanging magnetic field in the x-direction ($\vec{B} = B\hat{x}$). After a time t, what is the probability that it will have spin up in the z-direction.

You may take g_s to be exactly 2.

6. (15 pts). Consider a photon gas in equilibrium at temperature T inside a container of volume V.

a) (2 pts) Is the photon gas described by the Fermi-Dirac or by the Bose-Einstein statistics? Why? What is its chemical potential? Why?

b) (5 pts). Using the appropriate distribution function, derive the equation of state for the photon gas.

c) (3 pts). Compute the energy of a photon gas in terms of PV.

d) (5 pts). The ideal gas law states that $PV \sim NT$, where N is the number of gas molecules. Show that a similar expression can be written for a photon gas. What would replace N in the expression for a photon gas?

Note: You need not evaluate any integrals you might encounter in this problem.