Ph.D. QUALIFYING EXAMINATION
DEPARTMENT OF PHYSICS AND ASTRONOMY
WAYNE STATE UNIVERSITY

PART II

MONDAY, January 6, 2013
9:00 AM — 1:00 PM

ROOM 245 PHYSICS RESEARCH BUILDING

INSTRUCTIONS: This examination consists of six problems, each worth 10 points. Use a separate booklet for each problem. Write the following information on the front cover of each booklet:

1. Your special ID number that you received from Delores Cowen;
2. The problem number and the title of the exam (*i.e.* Problem 1, Part II).

Please make sure your answers are dark and legible.

Do NOT write your name on the cover or anywhere else in the booklet!

1. (**10 points**) Consider a system of a large number of distinguishable atoms N which are always at rest and have three non-degenerate energy levels, $-\mathcal{E}$, 0, and $+\mathcal{E}$. The system is in contact with a thermal reservoir at temperature T.

   (a) Compute the partition function for this system of N particles. (2 pts)

   (b) Compute the average internal energy per atom. (2 pts)

   (c) What is the average internal energy per atom in the limit of $T \to 0$ and $T \to \infty$? (2 pts)

   (d) Calculate the entropy per atom. (2 pts)

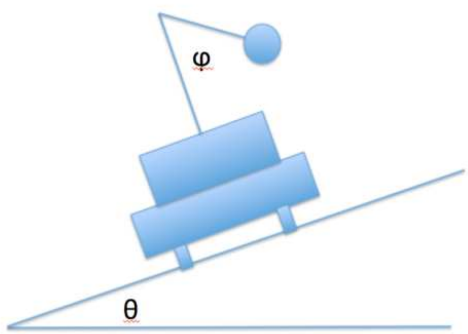
   (e) What is the entropy per atom in the limit of $T \to 0$ and $T \to \infty$? (2 pts)

2. (**10 points**) A car makes a turn on a road tilted by an angle θ. Inside the car there is a pendulum, which during the turn moves to an angle φ with respect to its support (see the figure).

   (a) Evaluate the angle between the pendulum and the vertical. (1 pt)

   (b) Evaluate the force of friction as a function of the given angles and the car weight W. (4 pts)

   (c) Evaluate the coefficient of static friction between the car and the road, if the car is barely able to complete the turn without skidding. (5 pts)



3. (**10 points**) A particle of mass m and charge q hangs from an ideal spring with spring constant k. It is displaced by a distance z_0 from rest and set into a state of small oscillations (along the vertical z-axis). The dipole electric and magnetic fields of a changing charge distribution at r, θ, ϕ (in spherical coordinates) are approximated as

$$\vec{E} = -\frac{\mu_0 p_0 \omega^2}{4\pi} \frac{\sin\theta}{r} \cos\left[\omega(t - r/c)\right] \hat{\theta}, \quad \vec{B} = -\frac{\mu_0 p_0 \omega^2}{4\pi c} \frac{\sin\theta}{r} \cos\left[\omega(t - r/c)\right] \hat{\phi},$$

where p_0 is the maximum dipole moment of the charge distribution at a given time t, ω is the frequency of small oscillations, r is the distance from the center of the dipole, μ_0 is the permeability of vacuum, and c is the speed of light in vacuum.

(a) Evaluate the Poynting vector as a function of the solid angle. (3 pts)

(b) Averaging over a cycle, evaluate the total power emitted. (4 pts)

(c) Using the result of (b), evaluate the time dependence of the maximum displacement of the particle $z_0(t)$. (3 pts)

4. (**10 points**) The Helmholtz free energy of a photon gas at temperature T inside a container of volume V is $F = -\alpha V T^4$, where α is a constant.

   (a) Using the above information, calculate the internal energy E of a photon gas and derive the relationship between E and PV, where P is the pressure of the gas. (3 pts)

   (b) Derive the average number of photons $\bar{N}_{ph}$ in a volume V at temperature T. You do not need to evaluate any integrals. (5 pts)

   (c) Using the results from (a) and (b), derive the equation of state for a photon gas in terms of P, V, and $\bar{N}_{ph}$ and compare it to the equation of state for a classical ideal gas. (2 pts)

5. (**10 points**) A particle is subject to a central force $F(r) = -k/r^{\alpha}$, where r is the radius of the particle orbit and k is a constant.

(a) Prove that the orbit must be circular if the particle energy is equal to its effective potential energy $V(r) = U(r) + l^2/(2mr^2)$, where l is the particle angular momentum and m is the particle mass. Find the value of the orbit radius r_0. (3 pts)

(b) Evaluate for which range of α the circular orbit is stable. (4 pts)

(c) Within the α range that you found, compute the frequency of small radial oscillations around the nominal circular orbit, when the particle is perturbed by a small radial displacement. (3 pts)

6. (**10 points**) The wavefunction of a particle is given by $\psi = A(x + 2z)\exp(-\alpha r)$, where $r = \sqrt{x^2 + y^2 + z^2}$ and α is a real constant.

(a) Find A. (3 pts)

(b) What are the expectation values of the orbital angular momentum operator L^2 and the z-component angular momentum L_z? (5 pts)

(c) If measuring L_z, what is the probability of getting a value of $+\hbar$? (2 pts)

Hint: The first few spherical harmonics are given by

$$Y_0^0 = \left(\frac{1}{4\pi}\right)^{1/2}, \quad Y_1^0 = \left(\frac{3}{4\pi}\right)^{1/2} \cos\theta, \quad Y_1^{\pm 1} = \mp \left(\frac{3}{8\pi}\right)^{1/2} \sin\theta e^{\pm i\phi},$$

$$Y_2^0 = \left(\frac{5}{16\pi}\right)^{1/2} (3\cos^2\theta - 1), \quad Y_2^{\pm 1} = \mp \left(\frac{15}{8\pi}\right)^{1/2} \sin\theta \cos\theta e^{\pm i\phi}.$$