Ph.D. QUALIFYING EXAMINATION
DEPARTMENT OF PHYSICS AND ASTRONOMY
WAYNE STATE UNIVERSITY

PART II

FRIDAY, JANUARY 6, 2012
9:00 AM — 1:00 PM

ROOM 245 PHYSICS RESEARCH BUILDING

INSTRUCTIONS: This examination consists of six problems each worth 10 points. Use a separate booklet for each problem. Write the following information on the front cover of each booklet:

1. your special ID number that you received from Delores Cowen,
2. the problem number and the title of the exam (*i.e.* Problem 1, Part II).

Please make sure your answers are dark and legible.

Do NOT write your name on the cover or anywhere else in the booklet!

1. **(10 points):** Consider a spin-1/2 electron in a hydrogen atom with orbital momentum $l = 1$.

   (a) What are the possible values of the total angular momentum J and projections of the total angular momentum on z axis? (1 pt.)

   (b) For all possible j and j_z derive the corresponding wave functions in terms of spherical harmonics Y_l^m and spinors $\chi_\pm$.

   Now, assume that the electron is in the state described by the following wave function:

$$R(r)\frac{1}{\sqrt{6}}\left(Y_1^0\chi_+ + \sqrt{5}Y_1^1\chi_-\right)$$

   where $R(r)$ represents the (normalized) radial part of the wave function. (5 pts.)

   (c) If one measures the z components of the total angular momentum, what are the possible outcomes? What are the corresponding probabilities? (1 pt.)

   (d) If one measures the total angular momentum, what are the possible outcomes? What are the corresponding probabilities? (3 pts.)

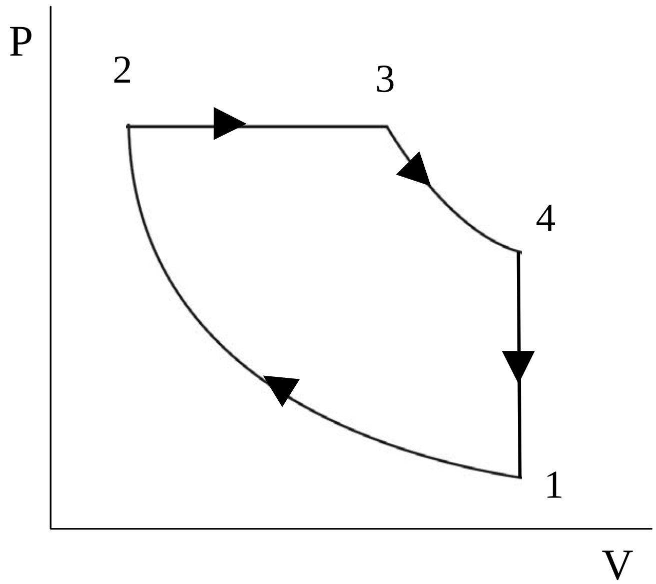
The raising and lowering operators for angular momenta could be useful:

$$J_+ |j, m\rangle = \hbar\sqrt{(j-m)(j+m+1)}\,|j, m+1\rangle$$

and

$$J_- |j, m\rangle = \hbar\sqrt{(j+m)(j-m+1)}\,|j, m-1\rangle\,.$$

2. **(10 points):** Calculate the efficiency of the Diesel cycle shown below, consisting of two adiabats, $1 \to 2$ and $3 \to 4$, one isobar, $2 \to 3$, and one constant volume process, $4 \to 1$. Assume C_P and C_V for the ideal gas. Express the answer in terms of the compression ratios V_3/V_1 and V_2/V_1.



3. **(10 points):** Two identical bosons (particles 1 and 2) are subjected to the one-dimensional harmonic oscillator potential

$$V(x_1, x_2) = \frac{1}{2} m\omega^2 (x_1^2 + x_2^2).$$

(a) For each particle, which is a 1D simple harmonic oscillator, the eigenfunction of the ground state can be written as

$$\psi_0(x) = A \exp(-bx^2).$$

Determine the constants A and b, and find the energy eigenvalue of the ground state. (3 pts)

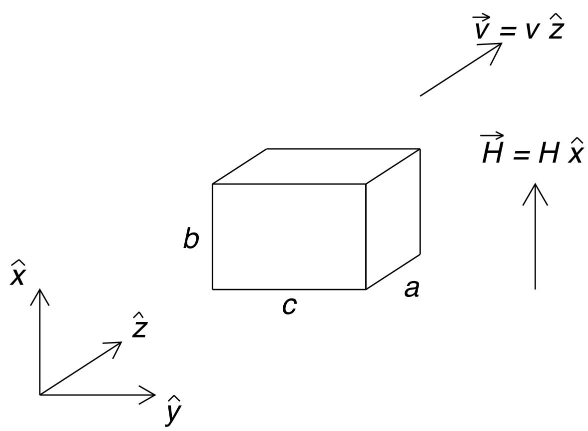
(b) What are the ground-state energy and wave function for this system of two identical bosons? (2 pts)

(c) If these two bosons interact with each other through the potential

$$V'(x_1, x_2) = \beta \exp(-a(x_1 - x_2)^2),$$

where $a > 0$, use perturbation theory to find the first order correction to the ground state energy. (5 pts)

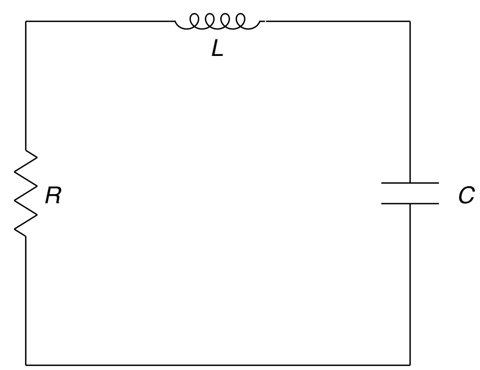
4. **(10 points):** An uncharged, rectangular, metal block has sides of length a, b, and c. It moves with velocity $\mathbf{v} = v\hat{\mathbf{z}}$ in a uniform magnetic field of intensity $\mathbf{H} = H\hat{\mathbf{x}}$. The edges of length a are parallel to the z axis, the edges of length b are parallel to the x axis, and the edges of length c are parallel to the y axis, as shown in the figure.



(a) What is the electric field inside the block? (6 pts.)

(b) What is the electric charge density in the volume and on the surface of the block? (4 pts.)

5. **(10 points):** A circuit consists of a resistor R, a capacitor C, and an inductor L connected in series, as shown below.



(a) Write down the differential equation for the charge flowing in the circuit. (2 pts.)

(b) Let $R = \sqrt{2L/C}$. Solve for charge on the capacitor and current in the circuit when, initially, the capacitor has charge Q_0 and there is no current. (7 pts.)

(c) Does the current oscillate? If it oscillates, at what frequency? If it doesn't oscillate, does the current change its direction of flow? (1 pts.)

6. **(10 points):** A block of mass m is given an initial velocity v_0 up an inclined plane. The coefficient of sliding friction between the block and the plane is $\mu = 0.1$. The inclined plane is fixed in position and has an angle of $\theta = 30°$.

   (a) How far up the plane does the block slide?

   (b) The block slides back down the inclined plane. How much time does it take for the block to slide up and back down to where it started from?