Ph.D. QUALIFYING EXAMINATION
DEPARTMENT OF PHYSICS AND ASTRONOMY
WAYNE STATE UNIVERSITY

PART I

WEDNESDAY, JANUARY 4, 2012
9:00 AM — 1:00 PM

ROOM 245 PHYSICS RESEARCH BUILDING

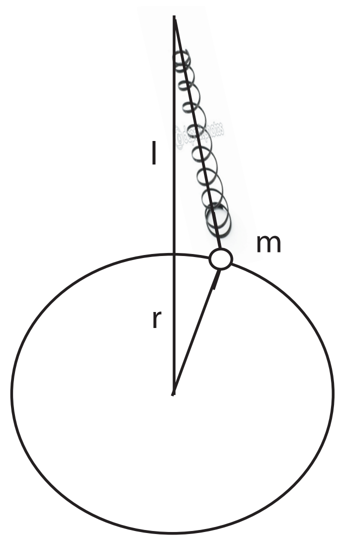
INSTRUCTIONS: This examination consists of six problems each worth 10 points. Use a separate booklet for each problem. Write the following information on the front cover of each booklet:

1. your special ID number that you received from Delores Cowen,
2. the problem number and the title of the exam (*i.e.* Problem 1, Part I).

Please make sure your answers are dark and legible.

Do NOT write your name on the cover or anywhere else in the booklet!

1. **(10 points):** A small bead of mass m can move without friction along a circular wire of radius r. A spring of length ℓ is attached to the bead. The spring exerts a force F on the bead. Assume that the force F is constant. Ignore gravity.



(a) How many degrees of freedom does this system have? (1 pt.)

(b) Write down the Lagrangian for this system. (7 pts.)

(c) Find the frequency of small oscillations, *i.e.* in the limit that the displacement of the bead is much smaller than ℓ, and r. (2 pts.)

2. **(10 points):** Consider a collection of identical noninteracting atoms, each of which has total angular momentum J. The system is in thermal equilibrium at temperature T and is in the presence of an applied magnetic field $\vec{H} = H\hat{z}$. The magnetic dipole moment associated with each atom is given by $\vec{\mu} = -g\mu_B\vec{J}$, where g is the gyromagnetic ration and μ_B is the Bohr magneton.

   (a) For an atom in this system, list the possible values of μ, the magnetic moment along the magnetic field direction and identify the magnetic energy corresponding to each state. (2 pts.)

   (b) Determine the mean value of the magnetic moment μ and the magnetization of the system M for $J = 1$. (6 pts.)

   (c) Find the magnetization of the system in the limits $H \to \infty$ and $H \to 0$ and discuss the physical meaning of the results. (2 pts.)

3. **(10 points):** A particle of mass m is confined in a one-dimensional square-well potential given by

$$V(x) = \begin{cases} -\alpha\delta(x - a/2), & 0 \leq x \leq a \\ \infty, & \text{otherwise} \end{cases}$$

where $\alpha > 0$.

(a) Solve the Schrödinger equation with appropriate boundary conditions and find the transcendental equation determining the allowed energies for $E < 0$. (6 pts)

(b) Sketch the graphical solution to the transcendental equation by plotting both sides of the equation on a figure. (1 pt)

(c) From the result of (a) and (b), determine the lower limit of α required for the existence of a solution for $E < 0$. (3 pts)

4. **(10 points):** Consider a spin-1/2 particle. Note that the Pauli matrices are given by

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \qquad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

(a) Find the eigenvalues and normalized eigenvectors (eigenspinors) of the spin operator $\hat{S}_y$. (3 pts)

(b) A measurement shows that this spin-1/2 system is in an eigenstate corresponding to the larger eigenvalue of the operator $\alpha\hat{S}_y + \beta\hat{S}_z$, where α and β are real constants. What is the eigenvalue and the corresponding normalized eigenspinor? (4 pts)

(c) Subsequently, another measurement of $\hat{S}_y$ is carried out. What is the probability that this measurement yields the result of $-\hbar/2$? (3 pts)

5. **(10 points):** An isolated conducting sphere of radius a has a charge $+Q$.

   (a) What is the capacitance of the sphere when it is infinitely far from other charges or objects? (3 pts.)

   (b) The same sphere is located a distance d from a grounded infinite conducting plane. If $d \gg a$, the capacitance of the sphere can be written as a power series expansion in a/d, and the first term is the value found in (a). Find the next (non-vanishing) term in the power series. (7 pts.)

6. **(10 points):** A long thin wire carrying a current I lies parallel to and at a distance d from a semi-infinite slab of iron, as shown below. Assuming the iron to have infinite permeability, determine the magnitude and direction of the force per unit length on the wire.