Ph.D. QUALIFYING EXAMINATION
DEPARTMENT OF PHYSICS AND ASTRONOMY
WAYNE STATE UNIVERSITY

PART I

WEDNESDAY, JANUARY 5, 2011
9:00 AM — 1:00 PM

ROOM 245 PHYSICS RESEARCH BUILDING

INSTRUCTIONS: This examination consists of six problems each worth 10 points. Use a separate booklet for each problem. Write the following information on the front cover of each booklet:

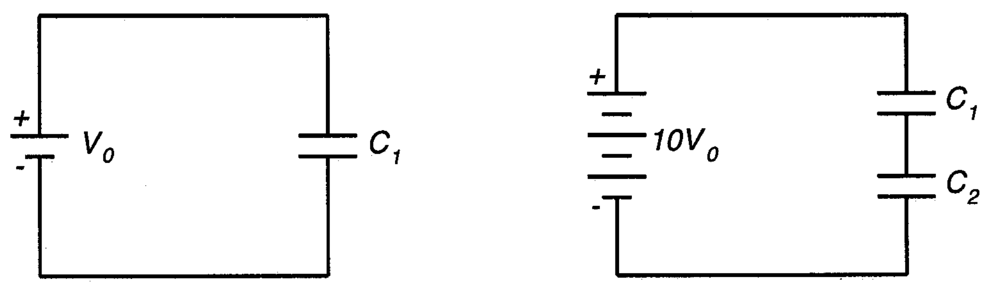
1. your special ID number that you received from Delores Cowen,
2. the problem number and the title of the exam (*i.e.* Problem 1, Part I).

Please make sure your answers are dark and legible.

Do NOT write your name on the cover or anywhere else in the booklet!

1. **(10 points):** You observe a binary star pair and measure the mean distance R between them and the period T of their approximately circular orbits.
   Use this information to determine the sum of their masses.

2. **(10 points):** A capacitor of capacitance $C_1 = C$ is charged by a battery of potential difference V_0. After fully charged, it is disconnected from the battery and reconnected in series to a second, uncharged capacitor of capacitance $C_2 = C/2$ and another battery of potential difference $10V_0$. The positive side of the first capacitor is connected to the positive terminal of the battery. Calculate the final potential across each of the capacitors.



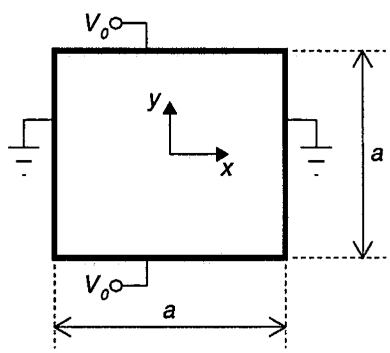
3. **(10 points):** A system at temperature T is composed of a very large number N of distinguishable non–interacting particles at rest. Each particle has two non–degenerate energy levels, 0 and $\epsilon > 0$.

   (a) (4 points) Find the average energy per particle, U/N. What is the maximum possible value of U/N for the system in thermal equilibrium at $T > 0$?

   (b) (4 points) Compute the entropy per particle as a function of $T > 0$.

   (c) (2 points) Determine the entropy in the limits of $T = 0$ and $T = \infty$ and comment on the results.

4. **(10 points):** A long, straight tube of square cross section has sides of length a. The tube is made of conducting material. The left and right sides of the tube are grounded, and the top and bottom sides are at potential $+V_0$. Using the x–y coordinate system indicated in the figure, find the potential everywhere inside the tube.



5. **(10 points):** It is well known that a particle of mass m in a one–dimensional δ–function potential has one and only one bound state. An impenetrable wall is placed near such a system so that the potential is given by

$$V(x) = \begin{cases} \infty & \text{for} \quad x < 0 \\ \alpha\delta(x-d) & \phantom{\text{for}} \quad x > 0 \end{cases}$$

Show that for a given value of $\alpha < 0$, the strength of the δ–function, the distance d needs to be smaller than d_0 to allow a bound state for the particle. Find d_0.

6. **(10 points):** Consider the spin state of an electron in a magnetic field. The Hamiltonian is

$$H = -\vec{\mu} \cdot \left(\vec{B}_d + \vec{B}_a\right).$$

Here $\vec{B}_d$ is a constant (dc) field pointing in the $+z$ direction, $\vec{B}_a$ is a time varying (ac) field given by

$$\vec{B}_a = B_0 \left(\cos \omega t \mathbf{i} + \sin \omega t \mathbf{j}\right),$$

and $\vec{\mu} = -C\vec{S}$ with $\vec{S} = (\hbar/2)\vec{\sigma}$, where $\vec{\sigma}$ are the Pauli spin matrices and C is a positive constant.

At time $t = 0$ the electron is in the spin down state. Find the probability of finding it in the spin up state at time $t > 0$.

Hint: Use the interaction picture. The Pauli spin matrices are

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Ph.D. QUALIFYING EXAMINATION
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PART II

FRIDAY, JANUARY 7, 2011
9:00 AM — 1:00 PM

ROOM 245 PHYSICS RESEARCH BUILDING

INSTRUCTIONS: This examination consists of six problems each worth 10 points. Use a separate booklet for each problem. Write the following information on the front cover of each booklet:

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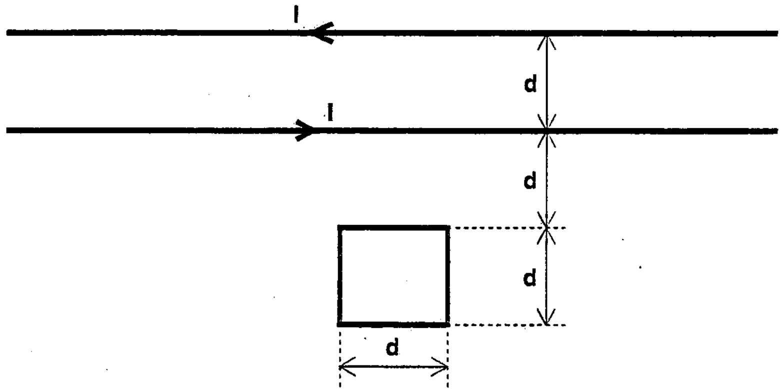
1. **(10 points):** Two infinite parallel wires separated by a distance d carry equal currents I in opposite directions, with I increasing at the constant rate $\alpha = dI/dt$. A square loop of wire with sides of length d lies in the plane of the wires at a distance d from the nearest parallel wire, as illustrated.



(a) (2 points) Is the induced current clockwise or counterclockwise? Justify your answer.

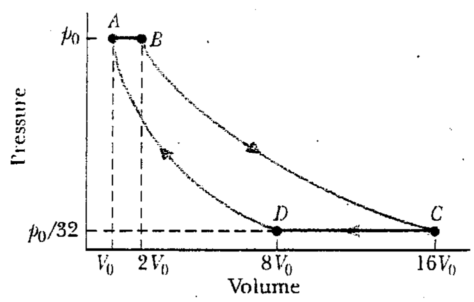
(b) (8 points) Find the emf induced in the square loop.

2. **(10 points):** An ideal gas is the working substance in an engine that operates on the cycle shown below. Processes BC and DA are adiabatic and reversible.

   (a) (3 points) Is the gas monatomic, diatomic, or polyatomic? Explain.

   (b) (4 points) What is the efficiency of the cycle shown?

   (c) (3 points) What is the best efficiency of any engine that operates within the temperature range used by the given cycle?



3. **(10 points):** A uniform bar of mass m and length $2a$ hangs horizontally by two parallel wires of length $2a$ attached to its two ends. The rod is suddenly given an angular velocity ω about a vertical axis through its center.

   (a) (4 points) Determine the distance h to which the bar rises.

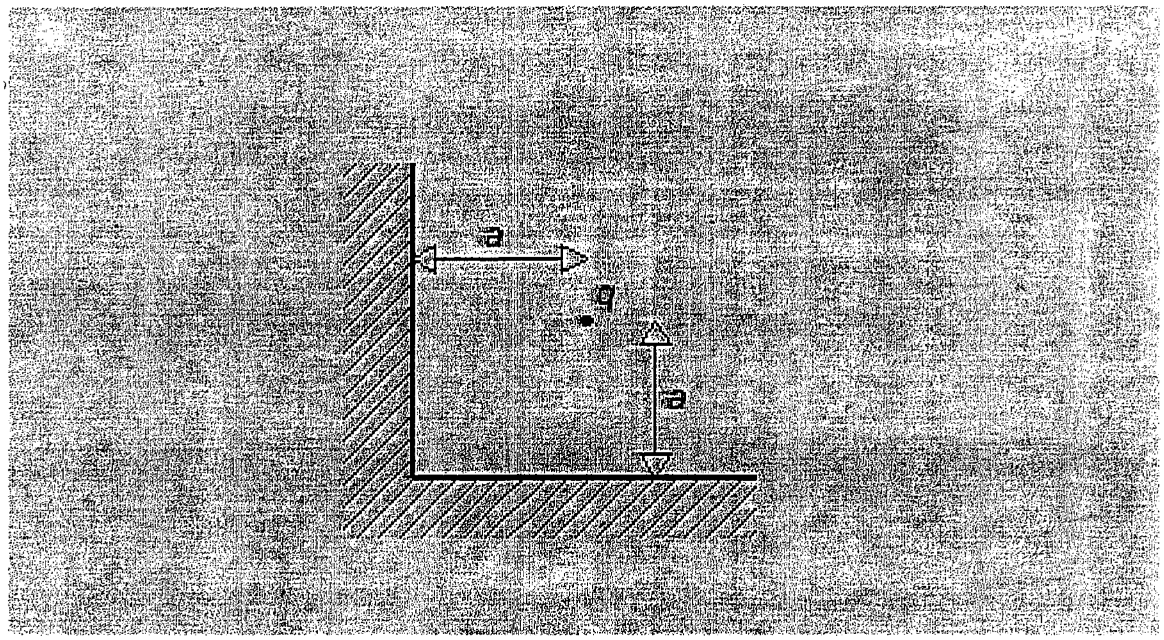
   (b) (2 points) Determine the relation between the height of the bar z and the rotation angle θ.

   (c) (4 points) Determine the initial increase in tension in each string.

4. **(10 points):** A point charge q is placed at a point located a distance a from each of two infinitely long conducting, grounded planes that intersect at 90 degrees. See the figure below.

   (a) (4 points) Compute the net force acting on the charge q.

   (b) (6 points) Compute the electrical potential energy of the system.



5. **(10 points):** Consider a *3-dimensional* harmonic oscillator described by the Hamiltonian

$$H_0 = \frac{\vec{p}^2}{2m} + \frac{1}{2}k\vec{r}^2$$

(a) (5 points) Give the energies for the ground state and the first excited state and their corresponding wave functions. Indicate if any of the states are degenerate.

(b) (5 points) A perturbation is applied with the form

$$H' = \alpha x$$

where α is a positive constant. Apply perturbation theory to calculate the correction to your eigen–energies for the ground and first excited states correct to first order in α.

Note: The ground state of the 1-dimensional simple harmonic oscillator has energy $E_0 = \frac{1}{2}\hbar\omega$ with $\omega = \sqrt{k/m}$. The corresponding eigenfunction is

$$\psi_0(x) = Ae^{-\frac{m\omega}{2\hbar}x^2}$$

where $A = (m\omega/\pi\hbar)^{1/4}$. The first excited state of the 1-dimensional simple harmonic oscillator has energy $E_1 = \frac{3}{2}\hbar\omega$ and eigenfunction

$$\psi_1(x) = \sqrt{\frac{2m\omega}{\hbar}}x\psi_0(x).$$

6. **(10 points):** Given two Hermitian conjugate operators a and $a^\dagger$ with $[a, a^\dagger] = 1$, let $N = a^\dagger a$.

(a) (5 points) Show that $[N, a^n] = -na^n$ and $[N, (a^\dagger)^n] = n(a^\dagger)^n$ where n is a positive integer.

(b) (5 points) Show that the only algebraic functions of a and $a^\dagger$ that commute with N are expressible as functions of N alone.