Ph.D QUALIFYING EXAMINATION
DEPARTMENT OF PHYSICS AND ASTRONOMY
WAYNE STATE UNIVERSITY

PART I

TUESDAY, JANUARY 5, 2010 9:00 A.M. - 1:00 P.M.

ROOM 245 PHYSICS RESEARCH BUILDING

INSTRUCTIONS: This examination consists of six problems, each worth of 10 points. Solve all six problems using a separate booklet for each problem. On the front cover of each booklet, write the following information:

1) your special ID number that you obtained from Delores Cowen
2) the problem number and the title of the exam (e.g., Problem 1, Part I)

Please make sure your answers are dark and legible.

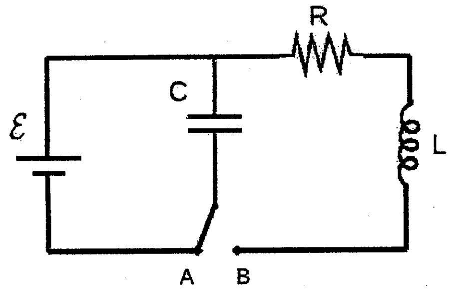
Do NOT write your name on the cover or anywhere else in the booklet!

Problem 1

In the electrical circuit shown below, $R = \sqrt{2L/C}$. After the capacitor has been charged from a battery of e.m.f. $\mathcal{E}$, the switch is moved from position A to B.

a) (5pts) Find the electric current in the resistor as function of time.

b) (5pts) Calculate the total energy dissipated in the resistor as a function of time (in time interval $[0, t]$), and compare its value in the large time limit to the initial energy stored in the capacitor.

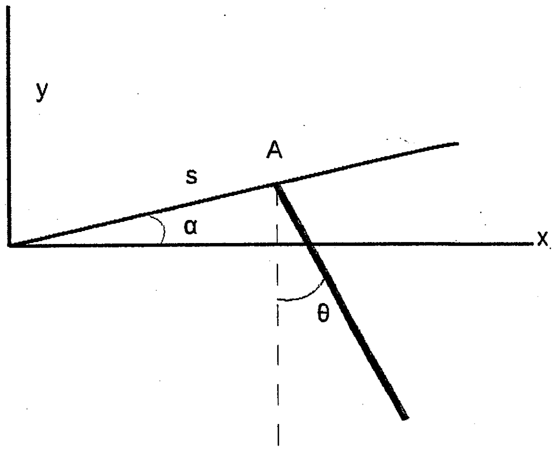


Problem 2

A uniform rod of length l and mass M moves in the vertical $x-y$ plane, subject to the force of gravity. One end point, A, is constrained to move in a straight line, given by the equation $y = x \tan\alpha$. See Figure.

a) (5 pts) Write the Lagrange's equations for the system using the coordinates $q_1 = s$ and $q_2 = \theta$. See Figure.

b) (5 pts) Find if a purely translational motion (θ =constant) is possible, and if so, for which value of θ.



Problem 3

Consider a particle of spin S in a constant uniform magnetic field **B**. The Hamiltonian that governs the dynamics of the spin state of the particle is

$$\hat{H} = -\hat{\vec{\mu}}\vec{B},$$

where $\hat{\vec{\mu}} = -\gamma\hat{\vec{S}}$. Here γ is named the gyromagnetic ratio.
Show that the expectation value of the observable $\vec{S}$, $\langle\vec{S}\rangle$, satisfies equation

$$\frac{d\langle\vec{S}\rangle}{dt} = -\gamma\ \langle\vec{S}\rangle \times \vec{B}$$

Problem 4

Find the condition for the energies at which particles with the mass m incident on a potential barrier

$$U(x) = \alpha[\delta(x) + \delta(x-a)]$$

are not reflected. Assume $\alpha > 0$ and being constant.

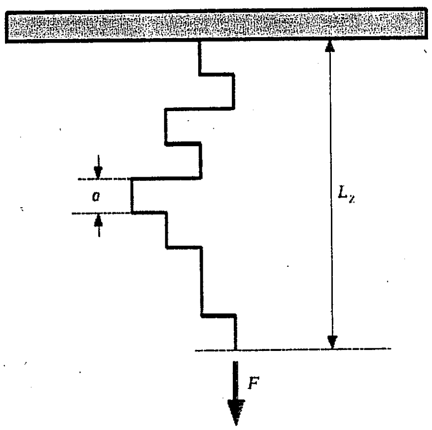
Problem 5

Consider a chain with $N \gg 1$ massless links of length a that can be oriented in three directions to the link above: left, right, or down, as shown below. Suppose that the upper end of the chain is fixed, a constant force F is applied to the lower end of the chain, and the system is in thermodynamic equilibrium at temperature T.

a) (4 pts) What is the mean end-to-end vertical extension, $\langle L_z \rangle$, of the chain?

b) (4 pts) Consider high and low temperatures limits, $Fa \ll kT$ and $Fa \gg kT$, respectively. Solve for $\langle L_z \rangle$, and estimate the entropy S of the chain in these limits.

c) (2 pts) Estimate the fluctuations in the mean vertical extension, $\langle (\Delta L_z)^2 \rangle = \langle (L_z - \langle L_z \rangle)^2 \rangle$, in the high temperature limit.



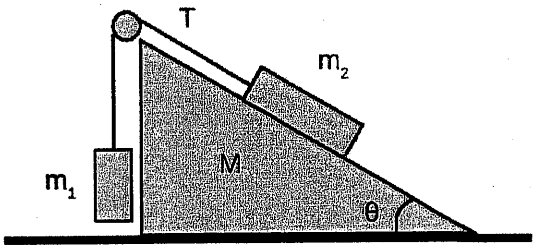
Problem 6

An inclined plane of mass M rests on a floor with friction coefficient μ. The incline makes an angle θ with the floor, and has a pulley attached at the top. Masses m_1 and m_2 are connected through a string, and the inclined plane surface is frictionless. See Figure.

a) (3pts) Assuming μ is very large, find the acceleration a of the two masses, and the tension T in the string.

b) (5pts) Find the minimum μ for which the incline remains at rest.

c) (2pts) Assuming $\mu = 0$, find the tension T in the string.



Ph.D QUALIFYING EXAMINATION
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PART II

THURSDAY, JANUARY 7, 2010 9:00 A.M. - 1:00 P.M.

ROOM 245 PHYSICS RESEARCH BUILDING

INSTRUCTIONS: This examination consists of six problems, each worth of 10 points. Solve all six problems using a separate booklet for each problem. On the front cover of each booklet, write the following information:

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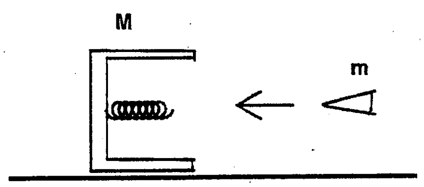
Problem 1

A bullet of mass m is shot at a target of mass M, which contains a spring of spring constant k. The bullet has initial velocity v and strikes the spring head on. The target is initially at rest and can slide without friction on the supporting plane. Find the maximal compression x of the spring.



Problem 2

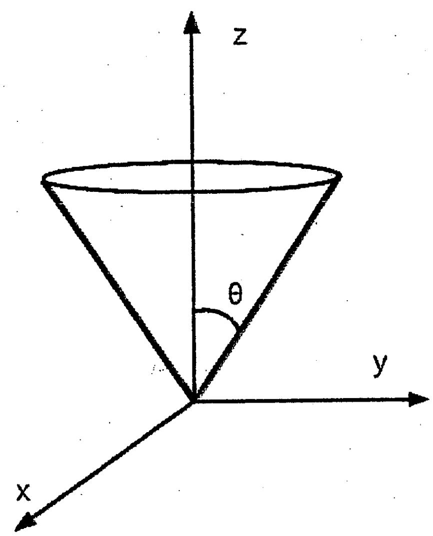
Consider a particle of mass m constrained to move on the inside surface of a cone with angle θ. The axis of the cone is parallel to the axis z as shown. The surface is frictionless and there is a uniform gravitation field g.

a) (3 pts) Construct the Lagrangian for the dynamics of the particle

b) (2 pts) Show that there are two constants of the motion

c) (2 pts) Show that it is possible to have the particle moving in a circular orbit, with radius $r = r_0$, where r is the distance to the z axis. Discuss the condition for this to happen.

d) (3 pts) Show that this orbit is stable with respect to small perturbations and find the frequency of small oscillations of r around r_0.



Problem 3

(a) (4 pts) Consider 1d quantum oscillator described by Hamiltonian

$$\hat{H} = \frac{\hat{p}^2}{2m} + \frac{m\omega^2 x^2}{2},$$

with eigenvalues $E_n = \hbar\omega(n + 1/2)$. Show that the only non zero matrix elements of x are

$$x_{n,n+1} = x_{n+1,n} = \sqrt{\frac{(n+1)\hbar}{2m\omega}}$$

(b) (3 pts) Using the results from the previous question, find the lowest order non-zero correction to the ground state energy level of 1d quantum oscillator for perturbation $V(x) = \alpha x^3$

(c) (3 pts) Now consider 2d quantum oscillator:

$$\hat{H} = \frac{\hat{p}_x^2 + \hat{p}_y^2}{2m} + \frac{m\omega^2(x^2 + y^2)}{2}.$$

Find the splitting of the energy levels $E = 2\hbar\omega$ due to perturbation $V = \beta\, xy$.

You might find useful to use the operators $\hat{a}_\pm = (\hat{p} \pm im\omega x)/\sqrt{2m}$.

Problem 4

Consider a gas of dipole molecules (of the mass m and dipole moment p) at temperature T.

(a) (4 pts) Show that the potential energy of two dipoles, $\mathbf{p}_1$ and $\mathbf{p}_2$, at a distance r is given by

$$U = \frac{1}{4\pi\epsilon_0}\left[\frac{(\mathbf{p}_1 \cdot \mathbf{p}_2)}{r^3} - \frac{3(\mathbf{r}\cdot\mathbf{p}_1)(\mathbf{r}\cdot\mathbf{p}_2)}{r^5}\right]$$

(b) (6 pts) Find the mean interaction energy of two molecules at a distance r, $\langle U(r)\rangle$, averaged over dipole directions. Assume $|U| \ll kT$.

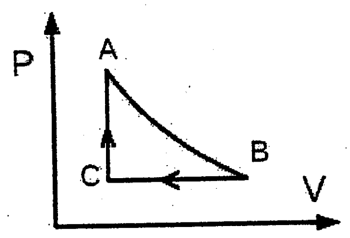
Problem 5

One mole of a diatomic ($\gamma = 7/5$) ideal gas is driven along a cycle depicted below. Process AB is an isotherm at temperature $T_A = 500$ K. Process BC is an isobar, and process CA is an isochore; $V_A = 1$ L, and $V_B = 4$ L. The gas constant $R = 8.31$ J/mol·K.

a) (3 pts) What is the pressure at point B?

b) (3 pts) What is the total work done per cycle?

c) (4 pts) What is the entropy difference $S_C - S_B$?



Problem 6

a) (5 pts) Find the charge distribution for which the electric field is given by

$$\mathbf{E} = \frac{q_0 \mathbf{r}}{4\pi\epsilon_0 r^3}\left[1 - \left(1 + \frac{r}{a} + \frac{r^2}{2a^2}\right)e^{-\frac{r}{a}}\right].$$

b) (5 pts) Find the magnetic field at the origin ($\mathbf{r} = 0$) for such a charge distribution rotating with angular velocity ω around z axis.