Ph.D. QUALIFYING EXAMINATION
DEPARTMENT OF PHYSICS AND ASTRONOMY
WAYNE STATE UNIVERSITY

EXAMINATION I
MECHANICS, THERMODYNAMICS AND STATISTICAL PHYSICS

MODAY, AUGUST 30, 2004
9:00 A.M. - 1:00 P.M.

ROOM 245 PHYSICS BUILDING

INSTRUCTIONS: This paper consists of four problems. You are to solve all four using a separate booklet for each problem. On the front cover of each booklet, you must write the following information.

1) Your special ID number that you obtained from Delores Cowen
2) The problem number and the title of the exam (i.e. Problem #1, M,THERMO.,STATS. Physics).
3) **Please press hard and make your answers legible to read.**

You must **NOT** write your name on the cover or anywhere else in the booklet!

EXAM 1, Problem 1

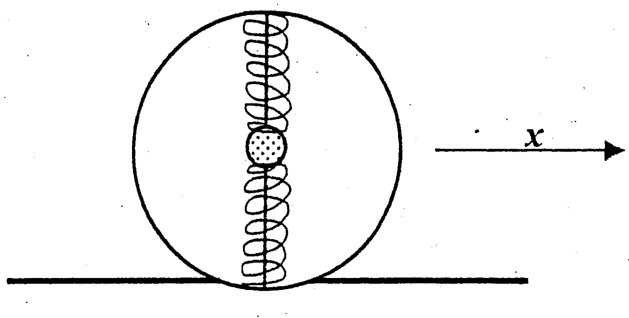
A rescue plane moving horizontally with a speed v_0 drops a package of supplies to a stranded polar expedition. The package is dropped from the plane at a height H.

a) (5 points) Determine the height y of the package as a function of horizontal distance x if the force due to air resistance is $\boldsymbol{F}_r = -\kappa \boldsymbol{v}$, where $\boldsymbol{v}$ is the velocity and κ is a positive constant.
b) (2points) How high should the plane fly in order to drop the package at a distance x_0 from the drop point in the absence of air resistance?
c) (3 points) Determine the first non-vanishing correction to the height obtained in part b) assuming that κ is small.

EXAM 1, Problem 2

A circular hoop of mass M and radius R is constrained to roll without slipping in the **x**-direction along a horizontal plane. The plane of the hoop is vertical. A bead of mass m and negligible diameter (diameter $<< R$) is constrained to move without friction over a thin straight wire attached along a diameter of the hoop. The bead is located between two identical springs each with a force constant k. The equilibrium length of each spring is equal to R. One end of each spring is attached to the hoop; the other end is attached to the bead (see Figure).

a) (4 points) How many degrees of freedom does this system have? Construct the Lagrangian for the system using appropriate variables.
b) (3 points) Find a (static) equilibrium configuration. Write the equations of motion describing small oscillations around the equilibrium point.
c) (3 points). Solve these equations to obtain the frequencies of the normal modes, and describe the normal modes.



*EXAM 1, **Problem 3***

An ideal gas of atoms of number density n at an absolute temperature T is confined to a thermally isolated container that has a hole of area A in one side of the container. Assume a Maxwell-Boltzmann velocity distribution for the atoms. The size of the hole is much smaller than the mean free path of the atoms which, in turn, is much smaller than the size of the container.

a) (4 points) Calculate the number of atoms escaping through the hole per unit time (express your answer in terms of the mean speed of the atoms in the container).
b) (4 points) Assume there is no back flow to the container. What is the ratio of the average kinetic energy of atoms leaving the container to the average kinetic energy of atoms initially occupying the container?
c) (2 points) Can you explain the result of part b) qualitatively?

EXAM 1, Problem 4

The nuclei of certain crystalline solids have spin 1. Each nucleus can be in any of the three states labeled by a number m, where $m = 1, 0, -1$. The nucleus has the same energy ϵ ($\epsilon > 0$) in the states $m = 1$ and $m = -1$ and zero energy in the state $m = 0$.

a) (4 points) Find an expression for the partition function for N such nuclei at an absolute temperature T.
b) (3 points) Compute the entropy of the system from the partition function found in a).
c) (3 points) Consider the entropy in the limits $T \rightarrow \infty$ and $T \rightarrow 0$. Explain your results.

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EXAMINATION II
E & M

WEDNESDAY, September 1, 2004
9:00 A.M. - 1:00 P.M.

ROOM 245 PHYSICS BUILDING

INSTRUCTIONS: This paper consists of four problems. You are to solve all four using a separate booklet for each problem. On the front cover of each booklet, you must write the following information.

1) Your special ID number that you obtained from Delores Cowen
2) The problem number and the title of the exam (i.e. Problem #2, E & M Physics).
3) **Please press hard and make your answers legible to read**.

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EXAM 2, Problem 1

A region between two parallel planes is filled with material with a permanent uniform magnetization **M**. Calculate the **B** field and the **H** field in all regions for the following two cases.

a) (5 points) The magnetization **M** is perpendicular to the plane.
b) (5 points) The magnetization **M** is parallel to the plane.

EXAM 2, Problem 2

Consider a simple device for the measurement of a static magnetic field. It consists of two co-axial metallic cylinders of radii a and b with $b > a$. A constant uniform magnetic field **B** is applied along the direction of the axis of the cylinders. A particle of mass m and positive charge q is introduced at the surface of the inner cylinder with zero kinetic energy. An electric potential difference $\Delta\phi$ is applied between the metallic cylinders such that the charged particle is attracted to the outer cylinder. If the potential difference is large enough, the charged particle can reach the outer cylinder, thus generating a current between the two cylinders.

Calculate the potential difference $\Delta\phi$, as a function of **B**, when this current starts to appear.

EXAM 2, Problem 3

Consider an electromagnetic plane wave propagating in a linear, isotropic and homogeneous medium with no free charge.

a) (2 points) Write down Maxwell's equations.
b) (3 points) Derive the wave equation that the electric field has to satisfy in a medium with permittivity ε, permeability μ, and conductivity σ.
c) (3 points) Show that a plane wave of angular frequency ω propagating along the +z direction will attenuate. Calculate the characteristic length of attenuation (the skin depth) in terms of the parameters ε, μ, σ, and ω.
d) (2 points) Estimate the skin depth using $(4\pi\varepsilon)^{-1} = 9 \times 10^{9}\ \mathrm{Nm^2/C^2}$, $\mu = 4\pi \times 10^{-7}$ Tm/A, and $\sigma = 10^{7}\ (\Omega\mathrm{m})^{-1}$ for frequency $f = 10^{15}$ Hz. Explain why a chunk of metal is opaque to visible light.

announced during exam: in part c, frequency → angular frequency; full credit is to be given to any who solved it before the announcement, assuming that ω is the frequency (in Hz) for their answer to part ©.

EXAM 2, Problem 4

A uniform charge of linear density λ is distributed on the z-axis.

a) (1 point) Calculate the electric field and the magnetic field everywhere.
b) (6 points) Calculate the electric field and the magnetic field measured by an observer moving with a uniform velocity $\boldsymbol{v}$ in the z direction with respect to the charged wire. Use a coordinate system that has the same orientation used in part a).
c) (3 points) How do your results of part b) compare with the fields generated by a uniform linear charge density λ and a current $-\lambda \boldsymbol{v}$ on the z-axis? Explain.

Possibly useful information

The electromagnetic-field tensor, $f_{\mu\nu}$, has the form:

$$f_{\mu\nu} = \begin{pmatrix} 0 & B_z & -B_y & -iE_x/c \\ -B_z & 0 & B_x & -iE_y/c \\ B_y & -B_x & 0 & -iE_z/c \\ iE_x/c & iE_y/c & iE_z/c & 0 \end{pmatrix}$$

and the current 4-vector has the form:

$$J_\mu = \begin{pmatrix} J_x \\ J_y \\ J_z \\ ic\rho \end{pmatrix}$$

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EXAMINATION III
QUANTUM

FRIDAY, September 3, 2004
9:00 A.M. – 1:00 P.M.

ROOM 245 PHYSICS BUILDING

INSTRUCTIONS: This paper consists of four problems. You are to solve all four using a separate booklet for each problem. On the front cover of each booklet, you must write the following information.

1) Your special ID number that you obtained from Delores Cowen
2) The problem number and the title of the exam (i.e. Problem #3, Quantum Physics).
3) **Please press hard and make your answers legible to read.**

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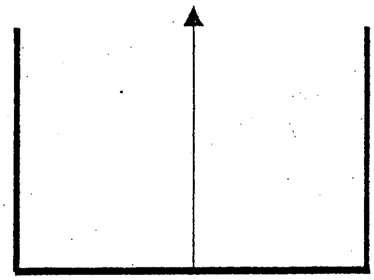
EXAM 3, Problem 1

Consider a particle in a square-well potential that has a delta function at its center:

$$V(x) = \begin{cases} \infty & x < 0 \\ g\delta(x-(a/2)) & 0 \le x \le a,\ g \ge 0 \\ \infty & x > a \end{cases}$$



a) (1 point) For $g = 0$, what are the eigenfunctions of the Hamiltonian and their corresponding energies? Which of the eigenfunctions are even and which are odd about the center ($x = a/2$)?

b) (4 points) Find an equation that can be solved for the energies of the states in part a) if g is no longer zero.

c) (3 points) Using the equation you found in part b), show which way the energy shifts (if at all) when g is no longer zero but is very small $\left(g \ll \frac{\hbar^2}{ma^2}\right)$ for the lowest even state and the lowest odd state. (A graphical solution is acceptable.)

d) (2 points) Sketch the wavefunctions of part c), indicating how they differ qualitatively from the shapes they would have for $g = 0$.

*EXAM 3, **Problem 2***

Consider the Hamiltonian of two identical non-interacting harmonic oscillators given by

$$H_0 = \frac{p_1^2}{2m} + \frac{p_2^2}{2m} + \frac{m\omega^2}{2}\left(x_1^2 + x_2^2\right).$$

a) (2 points) What are the energies and degeneracies of the two lowest-lying energy levels (*i.e.* the ground level and the first excited level)?

b) (4 points) Now suppose that the two oscillators are coupled so that the Hamiltonian is $H = H_0 + \lambda m\omega^2 x_1 x_2$, where λ is a dimensionless real number. For $\lambda << 1$, find the energy of this coupled two-oscillator system to first order in λ for each of the two lowest-lying levels.

c) (4 points) Find all of the exact eigenenergies of Hamiltonian H. Compare the exact energies for the states considered in part b) with the results obtained there.

Possibly Useful Information

$$\langle i|x|j\rangle = \sqrt{\frac{\hbar}{2m\omega}}\left(\sqrt{j+1}\,\delta_{i,j+1} + \sqrt{j}\,\delta_{i,j-1}\right)$$

$$\langle i|p|j\rangle = i\sqrt{\frac{\hbar m\omega}{2}}\left(\sqrt{j+1}\,\delta_{i,j+1} - \sqrt{j}\,\delta_{i,j-1}\right)$$

EXAM 3, Problem 3

Consider a Hamiltonian that has three possible eigenstates: $|\psi_1\rangle, |\psi_2\rangle$, *and* $|\psi_3\rangle$ with energies $E_i = \sqrt{(pc)^2 + (m_i c^2)^2}$, $i = 1, 2, 3$, where p is the momentum of the particle described by the Hamiltonian.

Suppose that the particles we can produce or detect are the following linear combinations of these eigenstates:

$$|\nu_e\rangle = \frac{1}{2}|\psi_1\rangle + \sqrt{\frac{3}{4}}|\psi_2\rangle \ , \ |\nu_\mu\rangle = \frac{3}{4}|\psi_1\rangle - \sqrt{\frac{3}{16}}|\psi_2\rangle - \frac{1}{2}|\psi_3\rangle \ ,$$

and $|\nu_\tau\rangle$ which is orthogonal to the other two detectable states.

If a $|\nu_e\rangle$ state is produced at $t = 0$,

a) (3 points) Find the probability that it will be detected as a $|\nu_e\rangle$ as a function of t.

b) (3 points) Find the probability that it will be detected as a $|\nu_\mu\rangle$ as a function of t.

c) (1 point) Show how your results in parts a) and b) determine the probability that it will be detected as a $|\nu_\tau\rangle$, and find that probability.

d) (3 points) For very large momentum, such that $\frac{(mc)^2}{p^2} << 1$, show that the probability that it will be detected as a $|\nu_\mu\rangle$ oscillates with a frequency proportional to $(m_1^2 - m_2^2)$ by finding that frequency.

EXAM 3, Problem 4

A beam of spin-1/2 particles of mass m, with momentum $\vec{p} = p\hat{z}$, scatters from a spin-dependent spherical potential:

$$\begin{cases} V = V_0 \sigma_z & r < R \\ V = 0 & r > R \end{cases} \quad \textit{where } V_0 \textit{ is a real positive constant.}$$

The energy of the particles is less than V_0 ($E<V_0$), and is small enough so that only s-wave scattering need be considered to obtain the cross section.

a) (1 point) Find a condition that the beam momentum must satisfy so that only s-wave scattering need be considered.
b) (4 points) Find an expression for the s-wave phase shift when the beam particles are polarized in the +z direction. Find the corresponding expression when they are polarized in the –z direction instead.
c) (1 point) Find the cross section when the beam particles are polarized along the +z-axis in terms of the s-wave phase shifts of part b).
d) (4 points) Now suppose that the particles in the beam are polarized along the positive x-axis. Find the cross section for the spin to be flipped to the –x direction in terms of the s-wave phase shifts of part b).

Reminder: The equation for s-wave scattering of spinless particles is:

$$\frac{d\sigma}{d\Omega} = |f|^2, \textit{ where } f = \frac{1}{k} e^{i\delta_0} \sin\delta_0 \textit{ is the coefficient of } \frac{e^{ikr}}{r} \textit{ in the scattered wave.}$$