PH.D. QUALIFYING EXAMINATION
DEPARTMENT OF PHYSICS AND ASTRONOMY
WAYNE STATE UNIVERSITY

EXAMINATION I
MECHANICS, THERMODYNAMICS AND STATISTICAL PHYSICS

MONDAY AUGUST 27, 2001
9:00 A.M. TO 1:00 P.M.

ROOM 245 PHYSICS BUILDING

INSTRUCTIONS: This paper consists of four problems. You are to solve *all four* using a *separate* booklet for each problem. On the front cover of each booklet, you must write the following information:

1) Your Greek letter ID that you obtained from Delores Cowen.
2) The problem number and the title of the exam (i.e., Problem #3, Quantum Physics).

You must *not* write your name on the cover or anywhere else in the booklets!

Exam 1, Problem 1

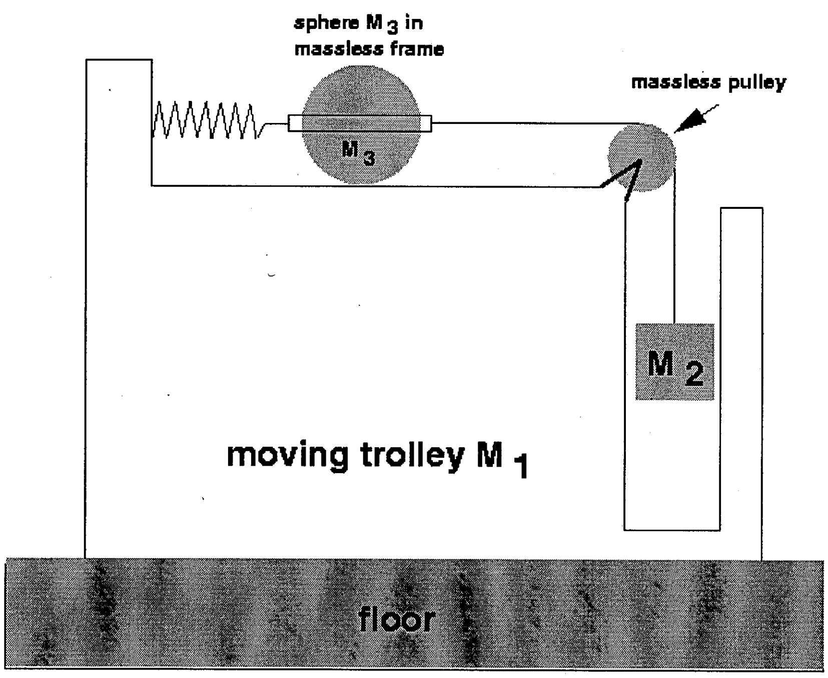
The drawing below shows a large trolley of mass M_1 and a weight of mass M_2 hanging from a rope that passes over a massless pulley and attaches to a sphere of mass M_3. The sphere is attached to the trolley by a spring of spring constant k. The sphere has radius R_3 and a mass density proportional to the radius squared. The spring is massless, the rope is massless and does not stretch, and the device that attaches them to the wheel is massless. The pulley rotates and the sphere rolls without slipping and without friction on the spindles. The mass M_2 slides straight up and down the shaft without friction.

(3 pts.) (a) Calculate the moment of inertia for the sphere with mass M_3 around its axis of rotation.

(3 pts.) (b) Determine the frequency of oscillations of the M_3-M_2 system. Assume that the trolley is rigidly fixed.

(4 pts.) (c) Determine the frequency of oscillations of the M_3-M_2 system if the trolley can slide without friction across the smooth floor.

Use the Lagrangian method for parts (b) and (c)



Exam 1, Problem 2

A block of mass M_1 moving with velocity v_0 on a frictionless air track strikes the first of two identical blocks with mass $M_2 = M_3$, connected by a massless spring with spring constant k. Consider the collision to be elastic and essentially instantaneous.

(5 pts.) (a) Find the oscillation frequency and amplitude of the spring system (M_2,M_3) in its center-of-mass frame.

(5 pts.) (b) Find an expression that relates the oscillation frequency to a minimum value of mass M_1 such that M_1 will hit the spring system (M_2,M_3) a second time.

Exam 1, Problem 3

A 'lattice gas' consists of N sites, each of which can be empty, or occupied by one particle in which case there is an extra energy ε associated with it. Each particle has a magnetic moment of magnitude μ that, in the presence of an applied magnetic field H, can adopt two orientations (parallel or anti-parallel to the field).

3 pts. (a) Find the canonical partition function for the system.

4 pts. (b) Find the average total energy of the system and its average total magnetization.

3 pts. (c) Find an expression for the fluctuations of the total energy of the system: $< (\Delta E)^2 >$, where E is the energy of the system.

Exam 1, Problem 4

Calculate the numerical values of the entropy changes for the following processes:

(2 pts.) (a) One mole of an ideal gas is expanded reversibly and isothermally to one-half its initial pressure.

(2 pts.) (b) One mole of an ideal gas is expanded at constant enthalpy to one-half its initial pressure.

(2 pts.) (c) One mole of an ideal gas undergoes a sudden free expansion to twice its initial volume.

(2 pts.) (d) One mole of H_2O at 90^oC is added to one mole of H_2O at 10^oC. Assume that the mechanical work done in mixing is negligible. Also neglect the heat capacities of the containers.

(2 pts.) (e) One mole of D_2O at 20^oC is added to one mole of H_2O at 20^oC.

$R = 8.31$ J mol/K, $C_p(H_2O) = 75.35$ J/K, $C_p(D_2O) = 83.72$ J/K

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EXAMINATION II
ELECTROMAGNETISM
(including OPTICS & SPECIAL RELATIVITY)

WEDNESDAY AUGUST 29, 2001
9:00 A.M. TO 1:00 P.M.

ROOM 245 PHYSICS BUILDING

INSTRUCTIONS: This paper consists of four problems. You are to solve *all four* using a *separate* booklet for each problem. On the front cover of each booklet, you must write the following information:

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Exam 2, Problem 1

A diffraction grating consists of 5 identical parallel slits. The width of each slit is b and the center-to-center spacing is a. The grating is illuminated by the collimated beam of a He-Ne laser of wavelength λ (633 nm) at normal incidence. The diffracted beam is projected to a screen at a distance L from the grating, again at normal incidence.

(2 pts.) (a) What relationship amongst L, a, b and λ constitutes the far field (also known as the Fraunhofer) condition?

(3 pts.) (b) Show that the far field intensity distribution is given by

$$I(\theta) = I_0\left(\frac{\sin(\beta)}{\beta}\frac{\sin(5\alpha)}{\sin(\alpha)}\right)^2,$$

Where I_0 is a constant and

$$\alpha = \frac{2\pi a}{\lambda}\sin(\theta); \qquad \beta = \frac{2\pi b}{\lambda}\sin(\theta).$$

(3 pts.) (c) Rederive the intensity distribution for the situation when the central slit is blocked off.

(2 pts.) (d) The ratio of intensities at $\theta = 0$ of the two cases above is expected to be 25/16. Give a simple explanation.

Exam 2, Problem 2

A capacitor consists of two coaxial pipes of radii a and b ($a > b$, a is the inner radius of the outer pipe and b is the outer radius of the inner pipe). The capacitor is lowered vertically into an oil bath of density ρ and dielectric constant κ.

(4 pts.) (a) Calculate the capacitance per unit length of this capacitor with and without oil between the pipes.

(6 pts.) (b) If $\kappa > 1$, show that the oil level between the pipes will rise higher than that outside. If a voltage V is maintained between the pipes, calculate the height difference.

Exam 2, Problem 3

Consider the propagation of electromagnetic waves along the z-axis in the vacuum space between two infinite parallel conducting plates located at $y = 0$ and $y = a$, respectively.

(2 pts.) (a) Write down, in components form, the differential equations and appropriate boundary conditions that the electric and magnetic fields, E_x, E_y, E_z, B_x, B_y and B_z, must satisfy.

(2 pts.) (b) Show that certain plane waves, restricted to the subspace $0 \leq y \leq a$, satisfy the conditions above. Find all such solutions and find for them the phase and group velocities in relation to their frequencies. (Since for these waves both the electric and magnetic fields are transverse to the direction of propagation, these waves are called transverse electric and magnetic or TEM modes.)

(6 pts.) (c) Find another class of solutions in which only the magnetic waves are transverse to the direction of propagation (the TM modes). For each mode give all components of electric and magnetic waves, the cut-off frequency, phase and group velocities and the time-averaged Poynting vector in terms of frequency and known constants.

Exam 2, Problem 4

Consider the pion production reaction, which can occur if a high energy cosmic ray proton collides with a stationary proton in the Earth's atmosphere,

$$p + p \rightarrow p + p + \pi^{+} + \pi^{-}.$$

In this problem, the goal is to find the minimum cosmic ray proton momentum such that this reaction can take place.

(5 pts.) (a) In the center of mass frame, find the minimum momentum in MeV/c of the incident proton that is just enough for the reaction to take place. The masses of the proton and pion are 938 and 140 MeV/c^2, respectively.

(3 pts.) (b) Let the velocity of the incident protons have magnitude v_{CM} in the center of mass frame. Show that, by using the Lorentz transformation law for energy and momentum, the incident proton has lab frame momentum magnitude,

$$|p_{L1}| = 2\beta\gamma E_{CM} / c$$

where E_{CM} is the energy of either of the protons in the center of mass frame and β and γ are the Lorentz factors associated with velocity v_{CM}.

(2 pts.) (c) Find the minimum momentum in MeV/c of the cosmic ray proton, in the lab frame, such that this reaction can take place.

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EXAMINATION III
QUANTUM PHYSICS

FRIDAY AUGUST 31, 2001
9:00 A.M. TO 1:00 P.M.

ROOM 245 PHYSICS BUILDING

INSTRUCTIONS: This paper consists of four problems. You are to solve *all four* using a *separate* booklet for each problem. On the front cover of each booklet, you must write the following information:

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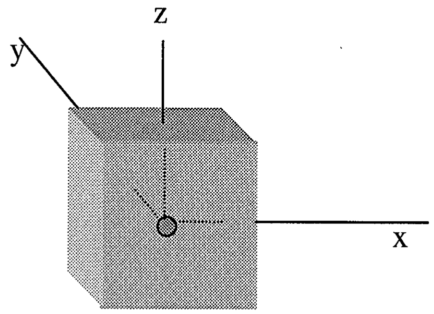
Exam 3, Problem 1



A particle of mass m is confined to a cube of side L. Inside the cube, there is a potential

$$V = V_0 \sin\left(\frac{\pi}{L}x\right)\sin\left(\frac{\pi}{L}y\right),$$

with the origin at the center of the cube and the axes parallel to its sides, as shown above.

4 pts. (a) Suppose $V_0 = 0$. Find the energies, degeneracies, and wavefunctions for the lowest and first excited energy levels of the system.

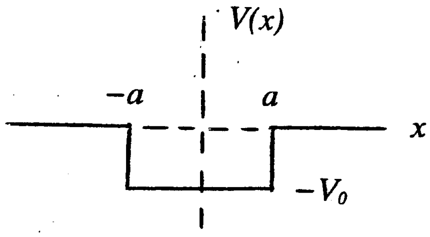
4 pts. (b) Treating V_0 as "very small," find the energies of the ground state(s) and first excited state(s) to first order in V_0. Also find the corresponding wavefunctions in the unperturbed limit.

2 pts. (c) In order to decide whether V_0 is "very small," to what quantity should it be compared, i.e., complete the expression $V_0 \ll$ ___________ .

Exam 3, Problem 2

The scattering of a low energy electron from a noble gas atom can be modeled in one dimension by a plane wave incident from the left on a square well potential. The potential is shown in the Figure and is specified by:

$V(x) = 0$	for	$x < -a$
$V(x) = -V_0$	for	$-a < x < a$
$V(x) = 0$	for	$x > a$



where V_0 is a positive number with units of energy and a is a positive number with units of distance. The electron is non–relativistic and has total energy E, with $E>0$.

4 pts. (a) Find an explicit representation for the time–independent wave function in each region that incorporates the appropriate boundary conditions.

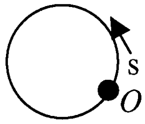
3 pts. (b) Show that the transmission coefficient T is unity when the DeBroglie wavelength λ inside the well is equal to $4a$. (In fact, $T=1$ for $\lambda = 4a/n$ where n is a positive integer. For this problem you only have to consider the case $n=1$.)

3 pts. (c) For the case, $V_0 = 5$ eV and $a = 1$ angstrom, find the value in eV of the lowest electron energy that satisfies the criterion in part (b). The mass of the electron is 0.511×10^6 eV/c^2 and Planck's constant is 4.136×10^{-15} eV–sec.

You may be interested to know that the phenomenon of lossless electron transmission is known as the *Ramsauer–Townsend effect.*

Exam 3, Problem 3

A particle of mass m is confined to a closed circular loop of wire of circumference c. Treat this as a one-dimensional problem, using the Schrödinger equation. Take the point O as the origin of the linear coordinate s measured counter-clockwise, whose range is 0 to c.



2 pts (a) Suppose that the potential along the circumference is even around O, i.e.,

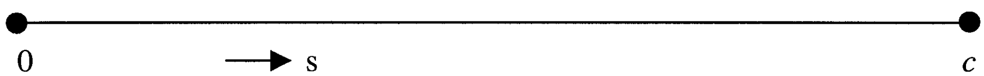
$$V(c-s) = V(s),$$

where inversion around O changes s to c-s because of the geometry of this problem. (Note that the momentum operator associated with s will behave the same way as it does under inversion where the position operator merely changes sign.)
Show that the stationary-state solutions can be chosen to be even or odd about the point O, as functions of s.

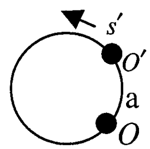
5 pts (b) Suppose that the potential along the circumference is zero, i.e.,

$$V(s) = 0.$$

Find all of the allowed energy levels for the particle of mass m. Find the corresponding wavefunctions that are even about O and those that are odd about O. Be sure to indicate the degeneracy of each energy level.
Sketch the wavefunctions that correspond to the lowest energy level and the first excited level, on lines that represents the linear distance from O as shown below:



3 pts (c) Suppose another observer comes along and, unaware of your calculation, uses the point O′ as the origin of the linear coordinate s' measured counter-clockwise, where the points O and O′ are separated by an arc of length a (as indicated in the diagram below). What results should that observer have obtained for the allowed energy levels and their corresponding even and/or odd wavefunctions about the point O′, as functions of s′? (No sketches required here.)
In general, how should that observer's wavefunctions, when expressed as functions of s, be related to those *you* found? Show that they are, indeed, related in that way.



Exam 3, Problem 4

A particle of mass M obeys the Schrödinger equation in 3 dimensions with a potential: $V(\vec{r})=\alpha r^2$, where α is a (real) positive constant.

4 pts. (a) Show that the angular dependence of the stationary-state wavefunctions may be chosen to be of the form: $\psi \propto f(\theta)\, e^{im\varphi}$, where θ is the (usual) angle made with the polar direction, φ is the (usual) azimuthal angle, and $f(\theta)$ obeys the equation:

$$\sin\theta \frac{d}{d\theta}\left(\sin\theta \frac{d}{d\theta} f\right) + \left[\kappa \sin^2\theta - m^2\right] f = 0,$$

where m and κ are constants.

You are *not* required to show that for acceptable solutions: $\kappa = \ell(\ell+1)$, where ℓ is an integer. The functions $f(\theta)$, are usually written as $P_\ell^m(\theta)$, which are called the Associated Legendre functions.

2 pts. (b) State (no derivation required) how ℓ and m are related to the angular momentum operators, and what their allowed values are? Show (no derivation required) the form of the angular momentum operator, in spherical coordinates, that corresponds to m and apply it to the wavefunction.

3 pts. (c) Show that the Schrödinger equation with this potential separates in Cartesian coordinates and find the ℓ and m values that correspond to the solution which has: $n_x = 1,\ n_y = 1$ *and* $n_z = 0$.

Note that the solutions to the 1-D Schrödinger equation with a harmonic oscillator potential are

$$\psi_n(x) = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4} \frac{1}{\sqrt{2^n\, n!}} H_n(\xi)\, e^{-\xi^2/2},$$

where n are integers, the H_n are Hermite polynomials and $\xi = \sqrt{\frac{m\omega}{\hbar}}\, x$.

1 pt. (d) Express the energy of the solution described in part (c) in terms of α, M and $\hbar$?

For Your Information

The lowest P_ℓ^m s are:

$P_1^0 = \cos\theta,\ P_1^1 = \sin\theta,\ P_2^0 = \tfrac{1}{2}(3\cos^2\theta - 1),\ P_2^1 = 3\sin\theta\cos\theta,\ P_2^2 = 3\sin^2\theta,$

and you may use the sign convention that yields $P_\ell^{-m} = P_\ell^m$.

The lowest Hermite polynomials are:

$H_0(\xi) = 1,\ H_1(\xi) = 2\xi,\ H_2(\xi) = 4\xi^2 - 2,\ H_3(\xi) = 8\xi^3 - 12\xi.$