Ph.D. QUALIFYING EXAMINATION
DEPARTMENT OF PHYSICS AND ASTRONOMY
WAYNE STATE UNIVERSITY

PART I: CLASSICAL MECHANICS

Wednesday, May 4, 2016
10 AM — 12 noon

Room 245, Physics Research Building

INSTRUCTIONS: This examination consists of three problems each worth 10 points. Use a separate booklet for each problem. Write the following information on the front cover of each booklet:

1. your special ID number that you received from Delores Cowen,
2. the exam and problem number (*i.e.* I.1).

Please make sure your answers are dark and legible.

Do NOT write your name on the cover or anywhere else in the booklet!

Problem I.1. Meteor Motion in the Earth's Atmosphere.

Consider a meteor in the Earth's atmosphere moving parallel to the Earth's surface at a low altitude - an altitude that is small compared to the Earth's radius. The meteor loses momentum as it collides with air molecules. In a simple model, the meteor can be treated as a circular disk that pushes the initially stationary air molecules so that they acquire the velocity and direction of the meteor at the time they are pushed. Assume that the downward velocity of the meteor is much smaller than its horizontal velocity and hence the downward motion can be ignored. The meteor has cylindrical radius R and mass M. At the altitude of the meteor, the atmosphere has volume mass density ρ.

a) (4 pts.) Find the momentum dp lost to the air in time dt.

b) (3 pts.) Using the result of a), find a differential equation relating the acceleration of the meteor and its instantaneous velocity.

c) (3 pts.) Solve the equation to find the time required to reduce the velocity by half.

Note of interest: The mass of a meteor can be estimated by observation of its instantaneous velocity and acceleration as found in b). The mass thus estimated is known as the "dynamical mass" and is accurate to roughly a factor of two.

Problem I.2.

Consider two identical beads of mass m, each carrying a charge q, constrained to move without friction on a horizontal wire ring of radius a.

a) (6 pts.) Find the Lagrangian and derive the equations of motion.

b) (2 pts.) Find a stationary solution.

c) (2 pts.) Consider a small perturbation to the stationary state and find the frequency of small oscillations.

Problem I.3.

Consider two particles each of mass m and charges $q_1 = -q_2 = q > 0$ that are connected by a spring of spring constant k. The particles and spring are located on a horizontal frictionless surface. The system is immersed in a strong uniform magnetic field of strength B oriented perpendicular to the surface. Neglect the electrostatic interaction of the particles between themselves and take into account only the interaction with the magnetic field and the spring. Use a Cartesian coordinate system with the magnetic field pointing along the positive z-direction. The particles are initially located at positions $(a, 0, 0)$ and $(-a, 0, 0)$. The particles are initially at rest with the spring stretched such that $2a > L_0$, where L_0 is the length of the unstretched spring.

a) (3 pts.) Draw a diagram indicating all forces acting on the particles (after they start to move) and show that the trajectory of the particles are symmetric with respect to the $y - z$ plane.

b) (4 pts.) Find the equations of motion (you might want to consider only the positively charged particle) and qualitatively describe the motion of the system.

c) (3 pts.) Solve the equations and find the frequency of oscillations if any.

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PART II: ELECTRICITY AND MAGNETISM

Wednesday, May 4, 2016
1:30 PM — 3:30 PM

Room 245, Physics Research Building

INSTRUCTIONS: This examination consists of three problems each worth 10 points. Use a separate booklet for each problem. Write the following information on the front cover of each booklet:

1. your special ID number that you received from Delores Cowen,
2. the exam and problem number (*i.e.* II.1).

Please make sure your answers are dark and legible.

Do NOT write your name on the cover or anywhere else in the booklet!

Problem II.1.

A point charge q is held at a point $D\hat{z}$ near a thin grounded infinite conducting plate, which occupies the x-y plane. Ignore gravity in this problem.

a) (1 pt.) Explain whether the charge feels an attractive or repulsive force with respect to the plate? HINT: use the method of images.

b) (3 pts.) Find the electric field in the positive-z and negative-z hemispheres, above and below the plate. Find the charge density on the plate.

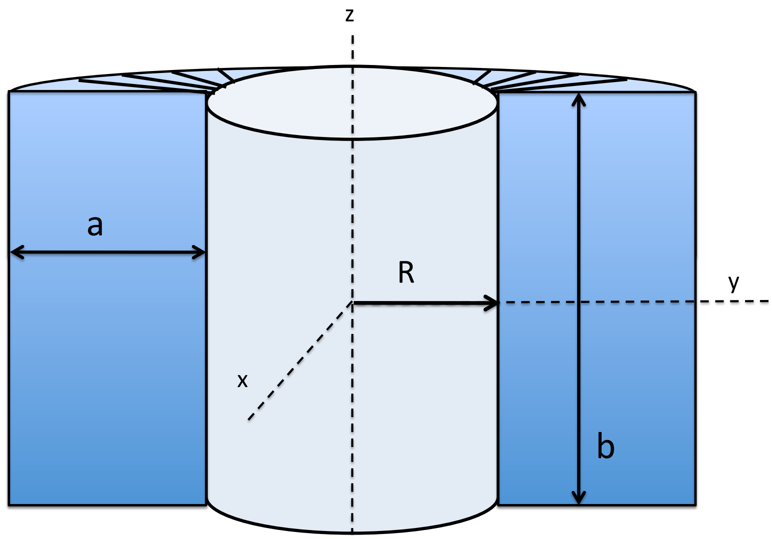
c) (2 pts.) Now consider another positive charge q placed at the point $-D\hat{z}$, the location of the first image charge. Make a sketch of the electric field in both hemispheres above and below the infinite grounded conducting plate.

d) (2 pts.) Find the charge density on the infinite conducting plate.

e) (2 pts.) Explain whether the net force experienced by the two charges is such that they are attracted towards each other and the plate, or they are repelled from each other and the plate. Find the magnitude of this force.

Problem II.2.

A toroidal inductor with inner radius R, has N turns of wire wound over a frame with a rectangular cross-section with a shorter side a and a longer side b, as shown in the Figure. The cylindrical space of radius R and height b, as well as the space within the toroid is filled with air, i.e., $\epsilon = \epsilon_0$, $\mu = \mu_0$.



a) (2 pts.) Assuming a steady current I through the coils, derive an expression for the B field within the toroid as a function of the radial distance ρ from the central z-axis and as a function of z. The origin of the coordinate system is at the center of the toroid.

b) (2 pts.) Find the self inductance L of this toroid. Assume all materials used to make the toroid have $\epsilon = \epsilon_0$ and $\mu = \mu_0$).

c) (2 pts.) Assume that this toroid is connected to a battery with voltage V_0 and resistance R, and that the resistance of the toroid is negligible compared to R. Find the potential drop as a function of time t across the inductor in terms of the L calculated in part b).

d) (4 pts.) Assuming that $b >> a$, find a general form for the induced electric field at time t at a point $(x, y, 0)$ inside the toroid. Use the two Maxwell equations $\nabla \cdot E = 0$ and $\nabla \times E = -\frac{\partial B}{\partial t}$. There is no need to evaluate any constants in the expression. A table of vector derivatives follows.

VECTOR DERIVATIVES

CARTESIAN. $d\mathbf{l} = dx\,\hat{\imath} + dy\,\hat{\jmath} + dz\,\hat{k}$; $d\tau = dx\,dy\,dz$

Gradient. $$\nabla t = \frac{\partial t}{\partial x}\hat{\imath} + \frac{\partial t}{\partial y}\hat{\jmath} + \frac{\partial t}{\partial z}\hat{k}$$

Divergence. $$\nabla\cdot\mathbf{v} = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}$$

Curl. $$\nabla\times\mathbf{v} = \left(\frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z}\right)\hat{\imath} + \left(\frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x}\right)\hat{\jmath} + \left(\frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y}\right)\hat{k}$$

Laplacian. $$\nabla^2 t = \frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} + \frac{\partial^2 t}{\partial z^2}$$

SPHERICAL. $d\mathbf{l} = dr\,\hat{r} + r\,d\theta\,\hat{\theta} + r\sin\theta\,d\phi\,\hat{\phi}$; $d\tau = r^2\sin\theta\,dr\,d\theta\,d\phi$

Gradient. $$\nabla t = \frac{\partial t}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial t}{\partial\theta}\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial t}{\partial\phi}\hat{\phi}$$

Divergence. $$\nabla\cdot\mathbf{v} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 v_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta\, v_\theta) + \frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial\phi}$$

Curl. $$\nabla\times\mathbf{v} = \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\theta}(\sin\theta\, v_\phi) - \frac{\partial v_\theta}{\partial\phi}\right]\hat{r} + \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial v_r}{\partial\phi} - \frac{\partial}{\partial r}(r v_\phi)\right]\hat{\theta} + \frac{1}{r}\left[\frac{\partial}{\partial r}(r v_\theta) + \frac{\partial v_r}{\partial\theta}\right]\hat{\phi}$$

Laplacian. $$\nabla^2 t = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial t}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial t}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 t}{\partial\phi^2}$$

CYLINDRICAL $d\mathbf{l} = dr\,\hat{r} + r\,d\phi\,\hat{\phi} + dz\,\hat{z}$; $d\tau = r\,dr\,d\phi\,dz$

Gradient. $$\nabla t = \frac{\partial t}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial t}{\partial\phi}\hat{\phi} + \frac{\partial t}{\partial z}\hat{z}$$

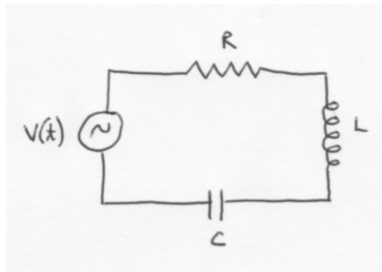
Divergence. $$\nabla\cdot\mathbf{v} = \frac{1}{r}\frac{\partial}{\partial r}(r v_r) + \frac{1}{r}\frac{\partial v_\phi}{\partial\phi} + \frac{\partial v_z}{\partial z}$$

Curl. $$\nabla\times\mathbf{v} = \left[\frac{1}{r}\frac{\partial v_z}{\partial\phi} - \frac{\partial v_\phi}{\partial z}\right]\hat{r} + \left[\frac{\partial v_r}{\partial z} - \frac{\partial v_z}{\partial r}\right]\hat{\phi} + \frac{1}{r}\left[\frac{\partial}{\partial r}(r v_\phi) - \frac{\partial v_r}{\partial\phi}\right]\hat{z}$$

Laplacian. $$\nabla^2 t = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial t}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 t}{\partial\phi^2} + \frac{\partial^2 t}{\partial z^2}$$

Problem II.3. Series RLC Circuit

Consider an alternating voltage source connected to a series R-L-C circuit as shown in the Figure. The voltage source provides a time-dependent voltage given by $V(t) = V_0\sin(\omega t)$ where V_0 and ω are real, positive numbers and t is time.



a) (4 pts.) Using Kirchoff's Law find a differential equation for the current $I(t)$ in terms of V_0, ω, t, R, L, and C.

b) (4 pts.) Assuming a solution of the form $I(t) = I_0\sin(\omega t + \phi)$, find I_0 and ϕ. Hint: use trigonometric identities so that the arguments of trigonometric functions are the same.

c) (2 pts.) Find the resonant frequency ω_0.