Ph.D. QUALIFYING EXAMINATION
DEPARTMENT OF PHYSICS AND ASTRONOMY
WAYNE STATE UNIVERSITY

PART III: QUANTUM MECHANICS

Friday, January 8, 2016
10 AM — 12 noon

Room 245, Physics Research Building

INSTRUCTIONS: This examination consists of three problems each worth 10 points. Use a separate booklet for each problem. Write the following information on the front cover of each booklet:

1. your special ID number that you received from Delores Cowen,
2. the exam and problem number (*i.e.* III.1).

Please make sure your answers are dark and legible.

Do NOT write your name on the cover or anywhere else in the booklet!

Problem III.1. A beam of particles can be described by a quantum-mechanical wave. Consider the 1-dimensional motion of a beam of particles of mass m and energy $E > 0$ traveling (non-relativistically) in the $-x$ direction and incident on a step potential at $x = 0$. (The particles have $x > 0$ before they reach the step.) The potential energy is described by

$$U(x) = \begin{cases} -U_0 & x < 0 \\ 0 & x \geq 0 \end{cases}$$

where U_0 is a positive real number.

a) (2 pts.) State the time-independent Schroedinger equation for each region of x.

b) (2 pts.) State the form of the solutions for each region of x.

c) (2 pts.) State the boundary conditions.

d) (3 pts.) Find the probability that a particle is back-scattered (reflected).

e) (1 pt.) Using the result in d) find the probability for back scattering in the classical limit.

Problem III.2. A one-dimensional quantum harmonic oscillator of mass m has ground state time-independent wavefunction $\psi_0(x)$ for potential energy $V(x) = \frac{1}{2}kx^2$ $(k > 0)$.

a) (6 pts.) Find $\psi_0(x)$ with the correct normalization factor using the lowering operator $\hat{a}$ for which $\hat{a}\psi_n(x) = \sqrt{n}\psi_{n-1}(x)$, where $\hat{a} = i\hat{p} + m\omega x$, $\omega = \sqrt{k/m}$, and $\hat{p}$ is the momentum operator. A mathematical table follows.

b) (4 pts.) At a certain point in time, k is instantly doubled so that the potential enmergy becomes $V_0' = kx^2$ for which the ground state is $\psi_0'(x)$. For the instant when V_0 is changed into V_0', find the numerical value of the probability of finding the particle in state ψ_0'.

Mathematical Formulas

Trigonometry:

$$\sin(a \pm b) = \sin a \cos b \pm \cos a \sin b$$

$$\cos(a \pm b) = \cos a \cos b \mp \sin a \sin b$$

Law of cosines:

$$c^2 = a^2 + b^2 - 2ab\cos\theta$$

Integrals:

$$\int x \sin(ax)\, dx = \frac{1}{a^2}\sin(ax) - \frac{x}{a}\cos(ax)$$

$$\int x \cos(ax)\, dx = \frac{1}{a^2}\cos(ax) + \frac{x}{a}\sin(ax)$$

Exponential integrals:

$$\int_0^\infty x^n e^{-x/a}\, dx = n!\, a^{n+1}$$

Gaussian integrals:

$$\int_0^\infty x^{2n} e^{-x^2/a^2}\, dx = \sqrt{\pi}\,\frac{(2n)!}{n!}\left(\frac{a}{2}\right)^{2n+1}$$

$$\int_0^\infty x^{2n+1} e^{-x^2/a^2}\, dx = \frac{n!}{2}a^{2n+2}$$

Integration by parts:

$$\int_a^b f\frac{dg}{dx}\, dx = -\int_a^b \frac{df}{dx} g\, dx + fg\Big|_a^b$$

Problem III.3. A spin 1/2 particle interacts with a magnetic field $\vec{B} = B_0\hat{z}$ through the Pauli interaction $H = \mu\vec{\sigma} \cdot \vec{B}$ where μ is the magnetic moment.

The Pauli spin matrices are $\vec{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$ where the σ_i are

$$\sigma_x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \quad \sigma_y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \quad \sigma_z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}.$$

The eigenstates for σ_z are the spinors

$$\alpha = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad \beta = \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

(a) (3 pts.) Suppose that at time $t = 0$ the particle is in an eigenstate χ_x corresponding to spin pointing along the positive x-axis. Find the eigenstate χ_x in terms of α and β.

(b) (7 pts.) For a later time t, find the probability that the particle is in an eigenstate corresponding to the spin pointing along the negative y-axis.

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PART IV: THERMAL, STATISTICAL, AND MODERN PHYSICS

Friday, January 8, 2016
1:30 PM — 3:30 PM

Room 245, Physics Research Building

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Problem IV.1. One mole of a van-der-Waals gas (at pressure P_i and temperature T_i) is held within a container with a movable piston. The walls of the container are adiabatic (cannot absorb any heat). A piston can be used to change the volume V of the gas in the container. There is a valve on the piston that when opened allows gas to freely flow through it. Initially the gas is only in the bottom portion of the container with a volume $V_0/2$.

a) (5 pts.) The valve is opened and the gas is allowed to expand into the remainder of the container (the full volume V_0). Assuming that the specific heat at constant volume c_V is independent of temperature, what is the new temperature of the gas?

b) (5 pts.) Then the piston is drawn completely to the top. The valve is now shut and the piston is pushed down until the entire gas is in the bottom of the container within volume $V_0/2$. What is the new temperature of the gas ?

The van-der-Waals gas has an equation of state

$$\left(P + \frac{a}{V^2}\right)(V - b) = RT \tag{1}$$

where a, b are small constants.

Maxwell's relation is

$$\left(\frac{\partial P}{\partial T}\right)_V = \left(\frac{\partial S}{\partial V}\right)_T . \tag{2}$$

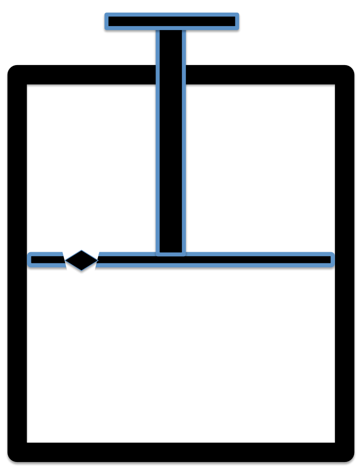

Figure 1: A gas is contained in a volume with a movable piston and a valve.

Problem IV.2. A box contains 4 distinguishable (classical) particles and 4 energy levels $0, a, 2a$ and $3a$ (where a is a positive constant). Ignore zero point energy in all calculations below.

a) (3 pts.) Initially the box is completely thermally insulated, with total energy $4a$ inside. What is the degeneracy of this state ?

b) (3 pts.) In the next scenario, the box is held in thermal contact with a heat reservoir at a temperature $T = 1/\beta$ (Canonical Ensemble). What is the partition function for the box? What is the mean energy at a temperature T of the reservoir? Find the temperature T_{4a} at which the mean energy of the box is equal to $4a$. Just set up the equation for this temperature, but do not solve it.

c) (4 pts.) Now, consider the case where the box in part a) is brought in contact with a reservoir at a temperature of T_{4a}. What is the mean energy transferred between the reservoir and the box? What is the variance (fluctuation) of this energy transfer?

Problem IV.3. A photon of energy E collides with a stationary electron whose rest mass energy is 511 keV. After the collsion the photon scatters at an angle θ (with respect to its original direction) with energy E'. The motion of the electron can be described classically (non-relativistically).

Answer the following questions in terms of the given variables and fundamental constants.

a) (1 pt.) What is the initial wavelength of the photon?

b) (2 pts.) What is the final-state (DeBroglie) wavelength of the electron?

c) (2 pts.) Suppose that the photon loses 100 eV of energy in the collision. Find the factor $\beta = v/c$ of the final state electron.

d) (5 pts.) Denote the ratio of E to electron rest mass energy by r and the ratio of E' to E by f. Find the scattering angle of the *electron* relative to the initial photon direction.