Ph.D. QUALIFYING EXAMINATION
DEPARTMENT OF PHYSICS AND ASTRONOMY
WAYNE STATE UNIVERSITY

PART II

Monday, May 11, 2015
9:00 AM — 1:00 PM

ROOM 245 PHYSICS RESEARCH BUILDING

INSTRUCTIONS: This examination consists of six problems, each worth 10 points. Use a separate booklet for each problem. Write the following information on the front cover of each booklet:

1. Your special ID number that you received from Delores Cowen;
2. The problem number and the title of the exam (*i.e.* Problem 1, Part I).

Please make sure your answers are dark and legible.

Do NOT write your name on the cover or anywhere else in the booklet!

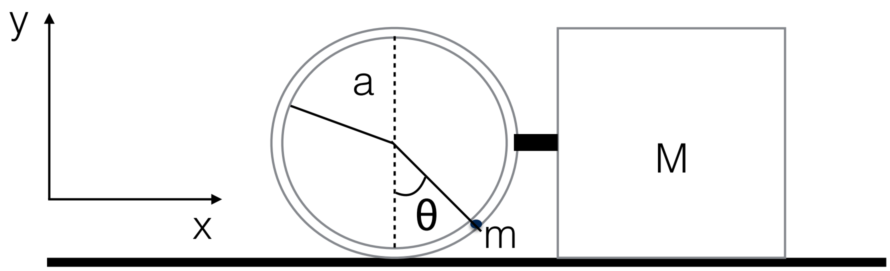
1. (**10 points**) Consider a gas of N particles in volume V in equilibrium with a thermal bath at temperature T.

   (a) (4 pts) Show that the system energy fluctuations can be expressed in terms of the temperature and the system's specific heat (at constant volume).

   (b) (4 pts) Evaluate the energy fluctuations explicitly in the case of an ideal monoatomic gas.

   (c) (2 pts) How the fluctuations change in the case of a diatomic gas?

2. (**10 points**) A block of mass M is rigidly connected to a massless circular track of radius a on a frictionless horizontal table as shown in the figure. A particle of mass m is confined to move without friction on the the vertical circular track.



a)(3 pts) Set up the Lagrangian, using θ as one coordinate.

b)(4 pts) Find the equations of motion.

c)(3 pts) In the limit of small angles, solve the equations of motion for θ and give the angular frequency of the oscillations of m.

3. (**10 points**) Consider a spin $s = 1/2$ particle in a constant uniform magnetic field of strength B_0 pointing along the z-axis.

   (3 pts) Find the eigenfunctions of the spin operator (for all three components)

   (3 pts) Find the eigenfunctions of the Hamiltonian and corresponding energy levels.

   (4 pts) Find the time dependence of the expectation values of spin projections on all three axes if the system is initially in the state with s_y=1/2.

4. (**10 points**) A 1 cm^3 metal cube at room temperature contains 2×10^{22} electrons.

a) (4 pts) Write down an expression for the density of states (DOS) $D(E)$. Evaluate the Fermi energy and thus prove that the system is a degenerate electron gas.

b) (2 pts) Find the Pauli paramagnetic susceptibility χ.

c) (2 pts) Find the pressure against the wall.

d) (2 pts) Find the average kinetic energy $< E >$.

5. Consider a long cylindrical wire of radius R carrying a current I flowing parallel to the direction of the axis of the wire. Within the wire the current density is j(r) , where r is the distance from the wire axis, and the direction is always parallel to the wire axis. This generates a magnetic field which may or may not cause a non-uniform charge density $\rho(r)$ in the wire. The wire is in equilibrium such that $j(r)$ and $\rho(r)$ do not depend on time and the drift velocity of the moving charges is a constant, v_D. Assume $\rho(r) = e\rho_L - e\rho_D$ where ρ_L is the uniform, fixed density of the unmoving, ionized atoms and $\rho_D(r)$ is the density of drift electrons (such that $j(r) = -e\rho_D(r)v_D$).

   a) (6 pts) Solve for $\rho_D(r)$ and $j(r)$. HINT: Use Gauss's Law and Ampere's Law noting the equilibrium state of the system.

   b) (4 pts) What is the stress on the wire due to the charge and current density? The expression for the EM Stress Tensor is

$$T_{ij} = \frac{\epsilon_0}{2}\left[E_i E_j - \frac{E^2\delta_{ij}}{2}\right] + \frac{1}{2\mu_0}\left[B_i B_j - \frac{B^2\delta_{ij}}{2}\right]$$

6. (**10 points**) Consider a charged particle in the ground state of the oscillator potential $U = m\omega^2/2$ under the influence of a weak electric field pointing along the direction of oscillation. Treating the interaction of the charge with electric field as perturbation,

(a)(3 pts) find the corrections to the energy level in the first order of perturbation theory.

(b)(4pts) find the corrections to the energy level in the second orders of perturbation theory.

(c)(3 pts) Compare with the exact solution.

You might find the following equations useful:

$$\hat{a}_{\pm} = \frac{1}{\sqrt{2\hbar m\omega}}(\mp i\hat{p} + m\omega x) \tag{1}$$

$$\hat{a}_{+}|n\rangle = \sqrt{(n+1)}\,|n+1\rangle;$$

$$\hat{a}_{-}|n\rangle = \sqrt{n}\,|n-1\rangle \tag{2}$$